Wake loss and energy spread factor of the LEReC Booster cavity caused by short range wake field

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LEReC project uses a DC photoemission gun with multi-alkali (CsK_{2}Sb or NaK_{2}Sb) cathode [1]. To get 24 mm “flat-top” distribution, 32 Gaussian laser bunches with 0.6 mm rms length are stacked together with 0.75 mm distance [2]. In this case one cannot simply use a 1 cm rms length Gaussian/step/delta bunch for short range wake field simulation since a 0.6 mm bunch contains frequency much higher than the 1 cm bunch. A short range wake field simulation was done using CST Particle Studio\textsuperscript{TM} with 0.6 mm rms Gaussian bunch at the speed of light, and this result was compared with the result for 1 cm rms Gaussian bunch in Figure 1, from where one notice that the wake potential for the 0.6 mm bunch is ~10 times higher than that of the 1 cm bunch. The wake potential of the 0.6 mm bunch, as well as the charge distribution, was then “shift and stack” every 0.75 mm, the normalized results are shown in Figure 2. The wake loss factor (WLF) is the integration of the product of wake potential and normalized bunch charge, and the energy spread factor (ESF) is the rms deviation from the average energy loss. It is calculated by summing the weighted squares of the differences and taking the square root of the sum. These two factors were then divided by $\beta^2$ for 1.6 MV beam energy. The wake loss factor is at 0.86 V/pC and energy spread factor is at 0.54 V/pC rms. With 100 pC electron bunch, the energy spread inter-bunch is 54 V rms.

Figure 1. Short range wake field of the Gaussian bunch with 1 cm (left) and 0.6 mm (right) length.

Figure 2. Short range wake field of the flat-top electron bunch, with wake loss factor at 0.86 V/pC and energy spread factor at 0.54 V/pC rms with 1.6 MV beam energy.
The $1/\beta^2$ factor mentioned above is a simplified estimation based on reference [3], in this paper we follow the method in [3] to get the wake loss and energy spread factors under different beam velocity. We start from the wake field of the 0.6 cm Gaussian bunch at the speed of light $c$, noted as $W_c(t)$, apply Fourier transform to it to get the frequency response $W_c(\omega)$, and then calculate the wake field at speed $v$ using $W_v(\omega) = W_c(\omega) / \left( I_0 \left( \omega R \sqrt{\gamma v} \beta \right)^2 \right)$, with $R$ the upstream beam pipe radius at 1.5 cm (downstream beam pipe is bigger, with radius at 4.0 cm), $I_0$ the modified Bessel function of the first kind of imaginary arguments, and $\beta=v/c$ [3]. Please note since the Fourier transform contains the components from negative frequency to positive frequency, the factor used in the above calculation should also be symmetric. Then inverse Fourier transform is applied to $W_v(\omega)$ to get the time domain response $W_v(t)$. This wake potential should contain real part only. Numerical calculation might give a small imaginary part that can be ignored, with its value less than 1% of the corresponding real part. $W_v(t)$ for different $v$ is shown in Figure 3. For comparison, the CST simulation results of the 33.3 pS (1, 0.9 and 0.8 cm for $\beta = 1, 0.9$ and 0.8, respectively) Gaussian bunch is shown in Figure 4, with a zoom-in plot between 6 and 7 nS. 0.6 mm Gaussian bunch was not simulated here since such a simulation is time consuming (~1 week with E5-2699 v3 @ 2.3 GHz 32 CPUs, 196 GB memory). The $I_0$ factor is always $\geq 1$ and is $\omega$ dependent, and the $\beta$ factor is always $\leq 1$ (in reality none of these two factors can be equal to 1). So the consideration of $1/\beta^2$ factor in the above paragraph gives WLF and ESF larger than those from the method in [3] that also considering the $1/I_0^2$ factor. Figure 3 showed a ~40% decrease of wake potential at 60~140 nS with $\beta$ drops from 1 to 0.9 or 0.8, and the correspondence number in Figure 4 is ~45% (at ~6.4 nS). Please note the space charge Coulomb effect is included in the CST simulations, and spikes at $t \approx 0$ pS in Figure 4 represent this effect, which is more severe at lower $\beta$ [4]. This effect was simulated by other codes and is not included in this paper.

Figure 3. Calculated wake potential of 0.6 mm Gaussian bunch with different velocity: $\beta=0.8$, 0.9 and 1, based on the CST simulation at $\beta=1$. 
Figure 4. Wake potential of 33.3 pS Gaussian bunch with different velocity: $\beta=0.8, 0.9$ and 1, CST simulation results.

The calculated wake potentials at different velocities were then “shift and stack” together to get the wake potential of “flat-top” electron bunch using the same method mentioned above. The results for $\beta=0.9$ and 0.8 are shown in Figure 5. Results of the WLF and ESF are shown in Table 1. While $\beta$ changing from 1 to 0.8, the WLF increases 44%, the ESF decreases 13%.

Figure 5. Short range wake field of the flat-top electron bunch for $\beta=0.9$ (left) and 0.8 (right).

Table 1. Wake loss factor (WLF) and energy spread factor (ESF) for different $\beta$.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>WLF [V/pC]</td>
<td>1.24</td>
<td>1.11</td>
<td>0.99</td>
<td>0.89</td>
<td>0.86</td>
</tr>
<tr>
<td>ESF [V/pC]</td>
<td>0.47</td>
<td>0.48</td>
<td>0.49</td>
<td>0.5</td>
<td>0.54</td>
</tr>
</tbody>
</table>

In this paper, we evaluate the short range wake field of the “flat-top” electron bunch in the LEReC booster cavity under different velocity. In the most severe case with $\beta=0.8$, the WLF is 1.24 V/pC and the rms ESF is 0.47 V/pC. For comparison, the 1 cm Gaussian bunch gives a WLF at 0.79 V/pC and an rms ESF at 0.31 V/pC. Space charge effect is not included in this calculation.

