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Determining the Ratio of the Horizontal β functions at BTA Multiwires MW006 and MW060 by Scanning the Emmitance

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During the most recent polarized proton run in January of 2001, the horizontal beam profile widths at BTA multiwires 6 feet (MW006) and 60 feet (MW060) downstream of the F6 septum were measured together at different horizontal emittances. The relationship between the β functions at the two multiwires was determined using this data. The horizontal trajectory of the beam injected into the Booster was adjusted to change its emittance.

Setup and Data Taking:

The last horizontal dipole in LTB, DH115, was varied to change the horizontal emittance. Three measurements at MW006 and MW060 of the horizontal beam width were taken for each setting of DH115. The current in DH115 was scanned from 83A to 218A. The beam intensity varied by a factor of 3 or 4 over this range due to the steering, but reasonable profiles were obtained in all cases. Thirteen sets of data were taken over this range. The parabolic fit feature of the BeamLineInstrument program was used to determine the Full Width at Half the Maximum value (FWHM) of the beam profile at both multiwires. Table I shows the data. The data is plotted in figure 1.

DH115 (A)	**MW006 FWHM (mm)**			**MW060 FWHM (mm)**		
82.9	6.57	6.56	6.55	25.69	25.55	26.45
98.9	6.63	6.63	6.60	27.80	27.66	28.49
107.9	7.40	7.44	6.72	28.45	29.43	28.33
117.9	7.61	7.57	7.51	31.59	31.39	33.31
127.8	7.76	7.73	7.51	33.91	33.63	33.99
137.9	8.52	8.49	8.50	37.44	37.57	36.58
147.8	8.67	8.71	8.70	39.86	39.76	39.98
157.7	8.87	8.91	8.95	42.45	42.89	42.54
167.8	9.74	9.19	9.18	44.18	44.31	45.29
177.7	10.00	9.99	10.02	47.61	47.50	45.85
187.8	10.59	10.08	10.09	47.00	48.16	49.75
197.7	10.70	10.71	10.74	51.52	51.92	52.05
207.8	10.81	10.79	10.80	54.63	53.12	53.09

Table I: BTA Multiwire Beam profile widths for different settings of LTB dipole DH115. These widths were obtained using BeamLineInstrument's parabolic fit algoritm. The data is arranged in ascending order with respect to the DH115 current.

Analysis:

The average of the three profile FWHM measurements for each value of DH115 was taken at both multiwires. This was divided by two to obtain the half width at half maximum at each multiwire for each value of DH115 (or emittance). The relation,

$$x^2 = \beta\varepsilon + d^2 , \qquad \text{Eq. (1)}$$

is used to find the MW060 β function from the half widths at both multiwires. 'x' is the measured half width value at either multiwire, 'β' is the value of the β function at either

multiwire, 'ε' is the emittance, and 'd' is the part of the beam width unaffected by the changes in emittance.

The value of β is relatively well known at MW006 since there are no BTA elements upstream of it. Its value was taken as equal to the MAD model value of 4.085 m.[1] Hence, the difference in the emittance for any two measurements is calculated using the equation,

$$\Delta\varepsilon_{i-j} = \frac{x6_i^2 - x6_j^2}{\beta_{006}}, \qquad \text{Eq. (2)}$$

where $x6_i$ and $x6_j$ are any two half width measurements i and j at MW006, $\Delta\varepsilon_{i-j}$ is the change in the (half-width) emittance for these two adjacent measurements, and β_{006} is the value of the β function there.

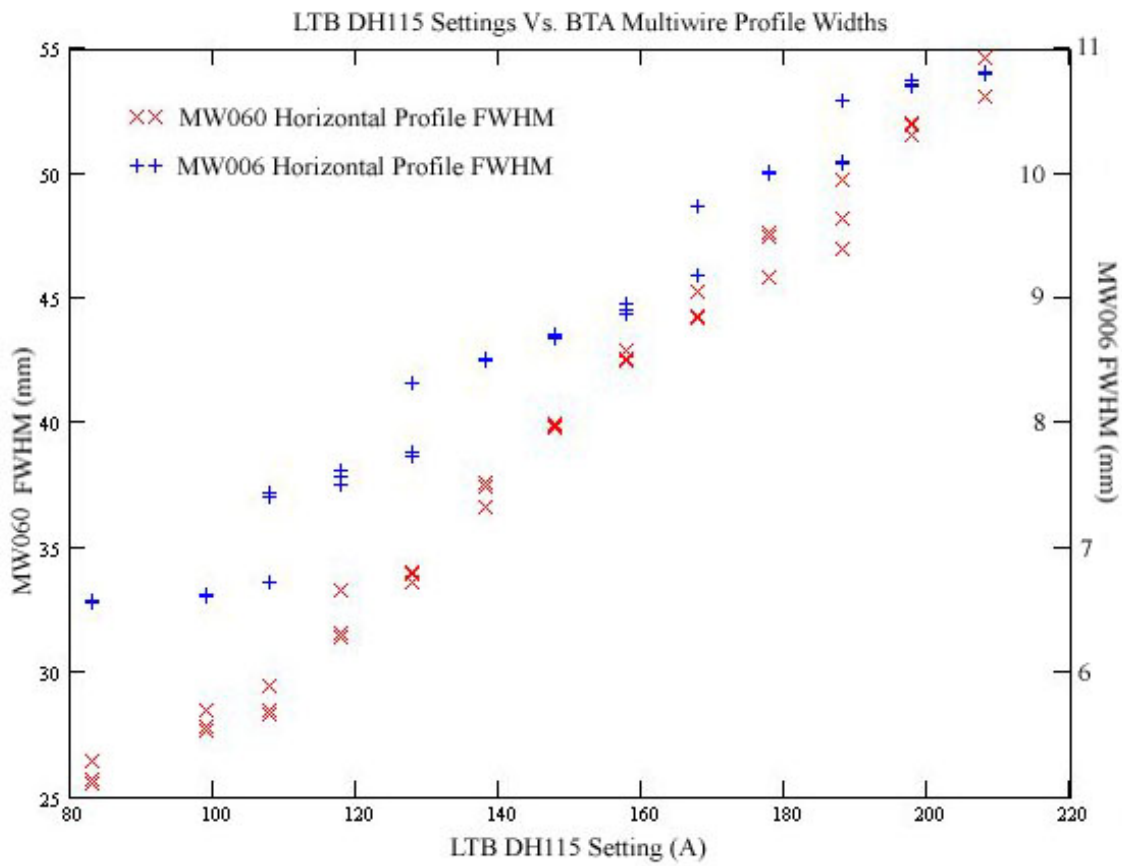

Figure 1: LTB DH115 Setting (A) versus BTA multiwire MW006 and MW060 beam profile full width at half Maximum (mm). Plot of data in table 1.

The transmission from MW006 to MW060 did not deteriorate much over the course of the measurements. When the beam size was small (low values of DH115), the ratio of the MW060 "Sum" (a measure of the beam intensity at the multiwire) over the Booster Late intensity was 12.55/400=0.031. When it was the largest the ratio was 3.2/120=0.027. Assuming there is perfect transmission between MW006 and MW060, the emittance at MW060 and MW006 should be the same. So, the value of the β function at MW060 should be given by,

$$\beta_{060} = \frac{x60_i^2 - x60_j^2}{\Delta\varepsilon_{i-j}}, \qquad \text{Eq (3)}$$

or more generally,

$$\frac{\beta_{060}}{\beta_{006}} = \frac{x60_i^2 - x60_j^2}{x6_i^2 - x6_j^2} \qquad \text{Eq. (4)}$$

Since these relations are true for any two measurements, two variables can be constructed from the measurements and plotted against each other to find β at MW060.

The measurements are arranged from the lowest (j=0) to the highest (j=12) value of DH115. The first variable is the sum of the changes in emittance ε^*_k from j=0 to j=k,

$$\varepsilon^*_k = \sum_{j=0}^{k} \Delta\varepsilon_{(j+1)-j}, \qquad \text{Eq. (5)}$$

ε^*_k is equal to the emittance plus an undetermined constant. The second variable is $X60^2{*}_k$,

$$X60^2{*}_k = \sum_{j=0}^{k} x60_{j+1}^2 - x60_j^2, \qquad \text{Eq. (6)}$$

It is the sum over the changes in the squares of the half widths of adjacent measurements, j+1 and j, at MW060. These two variables are plotted against each other in figure 2. Ideally, the data should have a linear dependence with a slope equal to the value of the β function at MW060,

$$X60^2{*} = \beta_{060}\varepsilon^*, \qquad \text{Eq. (7)}$$

If the MAD value of β_{006} is used, a linear fit of the data yields β_{060}=117 m. In general, the ratio β_{060}/β_{006} is 28.6. This method was also used to find β_{060} with the average width data plus and minus one standard deviation (assuming β_{006}=4.085m). In the average plus the standard deviation case a value of 121 m was obtained for β_{060}. In the other case, a value of 108 m was obtained. The MAD model value for β_{060} is 68.7m.

The values of the width at MW006 for the first two settings of DH115 are the same, while the values at MW060 are different (see table and figure 1). This may be because the parabolic fit has trouble when the beam is narrow. This can be seen from figure 3. Unfortunately, printouts of the profiles were not obtained during the study, so a detailed analysis of them cannot be made. However, if the data for the first two measurements is omitted from the analysis, the value obtained for β_{060} changes very little, it only goes up by about 3%.

The Relation Between the 95% Emittance and the FWHM Data

Using the parabolic fit data and equation 1, the 95% emittance can be calculated for each of the thirteen measurements as a function of 'd' at MW006 (d_{006}). The emittance has to be multiplied by βγ=2.4 to obtain the normalized emittance for the polarized proton BTA momentum.

For a Gaussian beam, the distance '*x*' from the center of the distribution where the beam intensity is e^{-n} of its peak value is related to '*f*', the fraction of the beam contained in *x-x'* phase space within '*x*' by $f=1-e^{-n}$.[2] For an '*x*' at which the intensity is half its peak value, $0.5=e^{-n}$. *f* happens to be equal to 0.5 as well (f=1-0.5). The following equation defines the emittance, or the area in x-x' phase space, that contains the fraction *f* of particles,

$$\varepsilon = \frac{2\pi\sigma^2}{\beta}\ln(1-f), \qquad \text{Eq. (8)}$$

where σ is the Gaussian standard deviation and β is the amplitude function at the location in question.[3] The 95% emittance would then be given by $6.0\pi\sigma^2/\beta$ and the emittance at half maximum by $1.4\pi\sigma^2/\beta$. The ratio of the two is 6/1.4=4.3. Hence, the 95% normalized emittance, εn_{95} is equal to $4.3\beta\gamma\varepsilon$, or $\varepsilon n_{95}=10.3\varepsilon$.

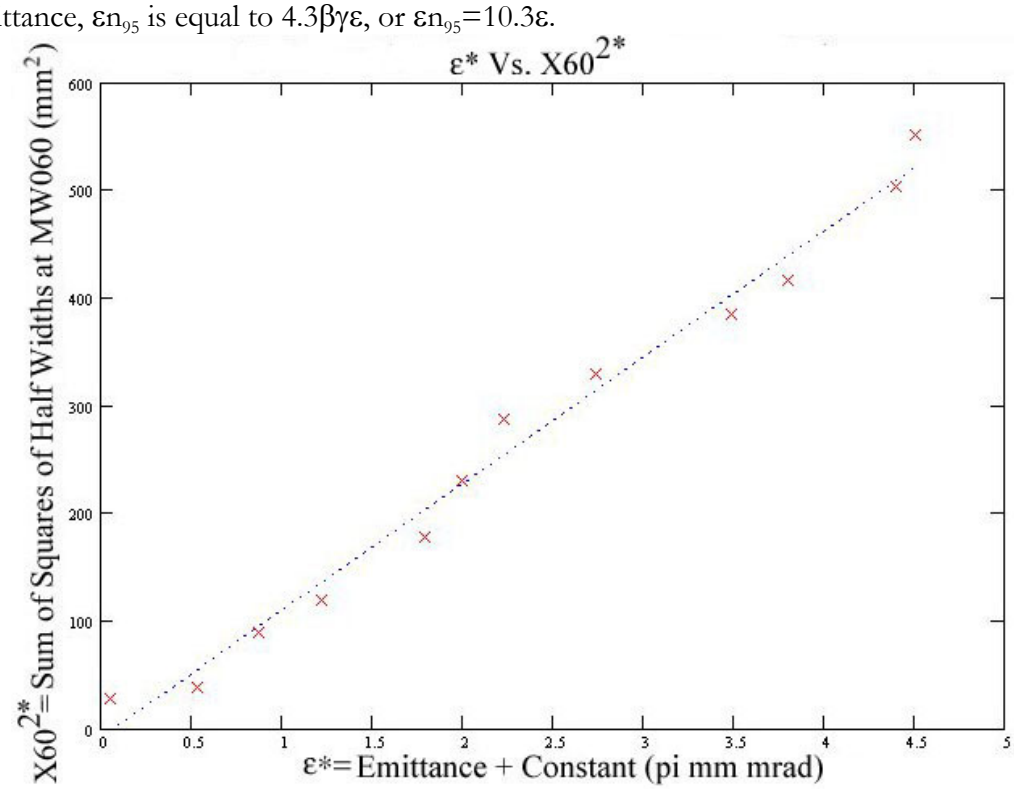

Figure 2: The sum of the changes in the half width emittance (ε^*_k) plotted against the sum in the squares of the half widths at MW060 ($X60^{2*}_k$). The slope of the linear fit is 117 m, which is identified with the β function at MW060.

There is not enough information to determine the values of the constant '*d*' in Eq. 1 for MW006 and MW060. Ideally, '*d*' should be the beam size's dispersion component. It might also be due to some kind of width measurement error that is independent of the beam size. The MAD model predicts a dispersion (D_{006}) at MW006 of –0.38 m. A value of 0.15% for the half width Δp/p was measured in the high intensity proton run immediately following the polarized run.[4] Using that value and a dispersion of -0.38 m gives a half width d_{006} of 0.6 mm. This value can be used to determine the 95% emittance using eq.1 and the MW006 data,

$$\varepsilon_{95} = \frac{4.3\left(x6^2\right) - d_{006}^2}{\beta_{006}}, \qquad \text{Eq (9)}$$

where the half width at half maximum has been multiplied by 4.3 to make it a 95% width. The relationship between d_{006} and d_{060} is then determined through,

$$d_{060}^2 = 4.3\left(x60^2\right) - \beta_{060}\varepsilon_{95}, \qquad \text{Eq. (10)}$$

with ε_{95} coming from equation 9.

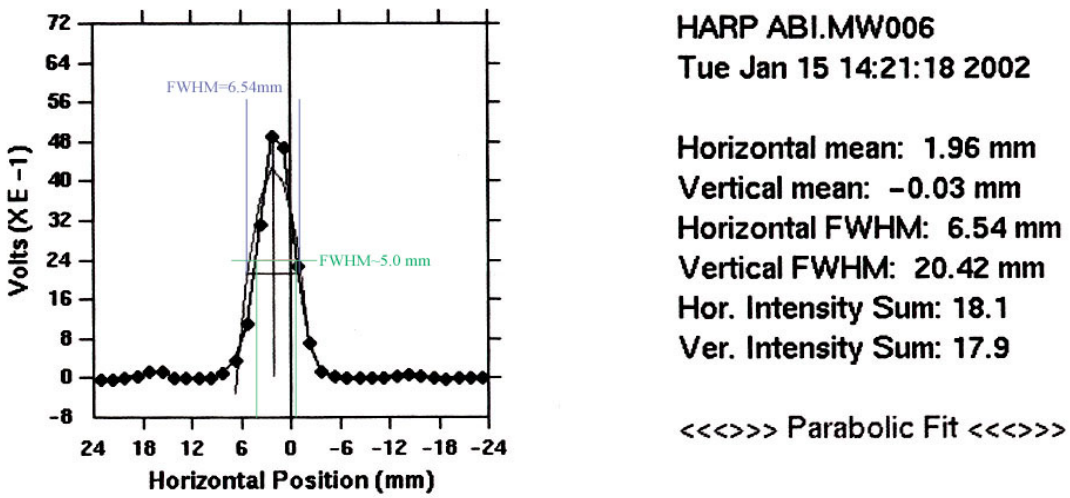

Figure 3: MW006 horizontal profile with parabolic fit for narrow beam. Note that FWHM from parabolic fit (6.54mm) is larger than that obtained visually from the graph (~5.0 mm). Wires are 1.5 mm apart.

Checking the Consistency Between the Dispersion Related Components of the Beam Size at the Two Multiwires

In order for d_{060} in equation 10 to be real for all thirteen measurements, the value of d_{006} must be greater than 5.1 mm (in order for it to be real for only one measurement it still has to be greater than 2.7 mm). So, there is an inconsistency between the value of d_{006} using the MAD model prediction (0.6 mm) and the value obtained for d_{060}. Given a half width $\Delta p/p$=0.15%, the absolute value of D_{006} must be greater than 3.4m in order for d_{060} to be real for all measurements. If D_{006} were equal to 3.4 m than the average value of D_{060} would be 6.2 m, which would give d_{060}=9.3 mm. The MAD model predicts that D_{060}=0.37 m.

If d_{006} equals 5.1 mm, εn_{95} ranges from 11.8 to 57.9 π mm mrad over the measurements. If d_{006}=0.6 mm, εn_{95} ranges from 26.8 to 72.9 π mm mrad. Assuming no emittance growth during acceleration, these values can be translated back to injection energy. They give an unnormalized maximum emittance of 83 π mm mrad in the d_{006}=5.1 mm case, and 106 π mm mrad in the d_{006}=0.6 mm case ($\beta\gamma$=0.69 at injection). If there is negligible coupling at injection, the horizontal acceptance will constrain how big the emittance can be (the vertical beam size at both multiwires did not change appreciably over the course of the measurements). The horizontal acceptance is limited at the beam dump to 203 π mm mrad, considerably larger than the unnormalized maximum emittance in either case.[5]

MW006 is often used to measure the emittance, particularly during polarized proton running. It is relevant what the value of d_{006} is, since the answer one gets for the emittance is highly dependent on it. The AGS IPM measurements indicate that it is difficult to reduce the horizontal εn_{95} below 20-30 π mm mrad, consistent with either value for d_{006}. These measurements require that d_{006} is larger than the MAD model predicts in order to have a real d_{060}, and so suggest that the horizontal εn_{95} is smaller than one finds when dispersion is neglected in calculating the emittance from the MW006 profile.

Given a value of d_{006}, varying β_{006} in equation 2 has no effect on the value of d_{060}. So, the requirement that d_{006} be large for d_{060} to be real does not depend on the value used for β_{006}. For a particular value of d_{006}, the value of d_{060} oscillates considerably around an average value over the range of measurements (the average value of d_{060} (D_{006}) is 10.5 mm (7m), and σ=4.9mm (3.3 m) for d_{006}=5.1 mm). However, the average value of d_{060}^2 does not become positive until d_{006} is greater than 4.75 mm.

Accuracy of the Parabolic Fit for Estimating the 95% Emittance

The 95% beam width as measured by counting wires can be compared to that calculated by multiplying the parabolic fit by $4.3^{1/2}$. Table II shows the results for the four profiles that are available.[6] It appears that, particularly in the MW060 case, that the approaches agree fairly well when the emittance is small, but the parabolic fit overestimates the 95% width when the emittance is large. This is clearest in figure 4 (bottom) for MW060, and is probably due to the fact that the shape of the beam is far from Gaussian when the beam is wide.

Profile	FWHM	Counting Wires	FWHM/2 x 2.07
MW006 (figure 3)	6.54 mm	6 mm	6.8 mm
MW006 (figure 5)	8.66 mm	7.5 mm	9.0 mm
MW060 (figure 4, top)	31.66 mm	32.5 mm	32.8 mm
MW060 (figure 4, bottom)	54.48 mm	41.3 mm	56.4 mm

Table II: 95% beam widths using FWHM parabolic fit x 2.07 and counting wires.

So, the parabolic fit is incorrect, possibly in some systematic but not constant way. With this in mind, the MW006 width data (Table I) is scaled with a factor of the form $^{M}/_{a}$+b (where a and b are constants and M is proportional to the difference in the DH115 setting from its lowest value). The parabolic fit technique starts out overestimating the 95% width by 6.8/6=1.13, and for a medium width overestimates it more (9/7.5=1.2).[7] Assuming this is a trend, since no profiles exist for the wide case, for every 1.5 mm increase the parabolic fit overestimates by 0.07. In that case, the last measurement would overestimate by a factor of 1.27. A similar situation exists for MW060, the narrow case is correct and the last case overestimates by a factor of 1.37. If the scaling is adjusted to fit these criteria, the value for β_{060} obtained is 103m. D_{060} is still imaginary when d_{006} is 0.6 mm, but has a smaller value than it does for the unaltered data (D_{060}=7.6*i* vs. 16.7*i* mm in the unaltered case). With this scaling, the 95% normalized emittance varies from 21 to 41 π mm mrad over the measurements.

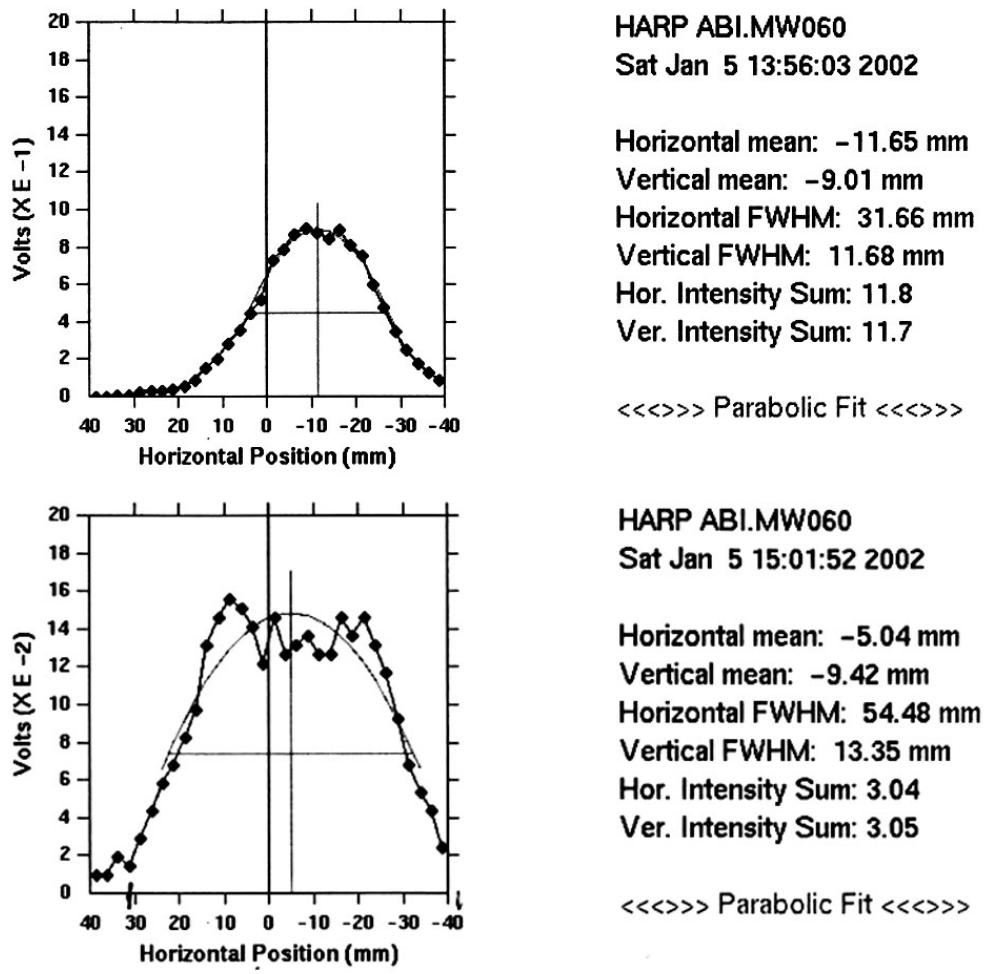

Figure 4: MW060 profiles with Parabolic fit for rather narrow (top, Horiz. FWHM=31.66 mm) and wide (bottom, Horiz. FWHM=54.48 mm) beam. Wires are 2.5 mm apart.

Additionally, the value of β_{060}, obtained from the unaltered MW006 data, does not vary much if only the first half (β_{060} =104m) or last half (β_{060}=115m) of the measurement set is used. If only measurements 4 through 9 are used β_{060}=131 m and d_{060} is imaginary when d_{006}=0.6 mm.

Summary

The ratio of the horizontal β functions at MW060 and MW006 was measured by scanning the BTA emittance and found to be 28.6. Assuming that the value of β at MW006 is that given by the MAD model, β_{006}=4.085m, this gives β_{060} equal to 117m. The MAD model predicts a value that is significantly smaller (β_{060}=68.7m). Additionally, the relationship between the values for the dispersion at MW006 and MW060 was investigated for consistency with the MAD model. If the value of D at MW006 is that given by the MAD model, D_{006}=-0.38m, and a nominal momentum spread is assumed, then the dispersion related beam width will be imaginary at MW060. A significantly larger absolute value for D_{006} (>3.4m) must be used for the data to yield a real value for the dispersion related beam width at MW060.

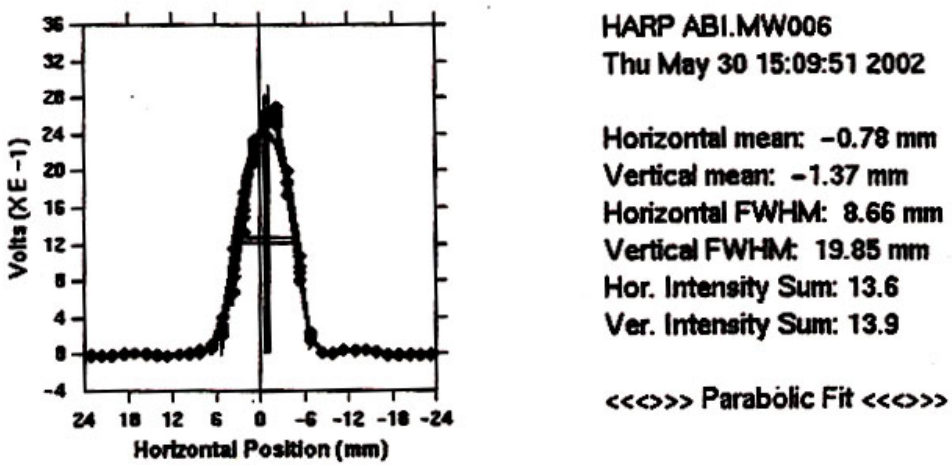

Figure 5:: MW006 horizontal profile with parabolic fit for medium width beam (FWHM=8.66 mm). This profile is from the high intensity proton run, NOT the polarized proton run. Wires are 1.5 mm apart.

At least in the MW060 case, the parabolic fit becomes a less accurate measure of the 95% width as the emittance is increased by missteering in LTB. This is likely because the beam profile becomes flatter and less Gaussian. The parabolic fit tends to overestimate the 95% width when the beam is wide (figure 4, bottom). Unfortunately, wide beam profiles do not exist for MW006, only the parabolic fit data does. Assuming a similar effect occurs with that multiwire, and the data is adjusted to compensate, the value for β_{060} obtained is roughly the same as for the unaltered data. In this case though, the value required for the dispersion at MW006 in order that the dispersion at MW060 will be real is not as large, but is still significantly larger than the MAD model prediction.

In hindsight, a significant shortcoming of the study was the lack of profile printouts. Without these printouts, particularly for MW006, it is difficult to verify the validity of using BeamLineInstrument's parabolic fit algorithm to analyze the data. This is particularly relevant in light of the discrepancy between the MAD prediction and the results obtained here for the values of the Beta and Dispersion functions at MW060.

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[1] Paul Sampson. All MAD model values used in the text (β_{006}, β_{060}, D_{006}, D_{060}) are from a BeamLineEmit MAD calculation using the Quad currents that were in use at the time and Polarized proton Kinetic Energy (1.49 GeV).
[2] W.T. Weng and S.R. Mane, "Fundamentals of Particle Beam Dynamics and Phase Space", pgs. 23-24, September 4, 1991.
[3] D.A. Edwards and M.J. Syphers, "An Introduction to the Physics of High Energy Accelerators", pg. 82, 1993.
[4] L.A. Ahrens. Booster Setup Book I, F.Y. 02, pg. 122.
[5] Booster Horizontal Acceptance value is from C. Gardner.
[6] The wider MW006 profile is from the high intensity proton run, a wide MW006 profile from the polarized proton run was not available.
[7] The medium width case is taken from the high intensity proton run. It is not the result of intentional injection missteering. So, unlike with missteered beam the profile is Gaussian. Given the supposition that the problem with the parabolic fit has to do with a non-Gaussian beam profile, any error here will be unrelated to it. However, with MW006 the parabolic fit seems to have a problem with a narrow beam that is unrelated to whether or not it is Gaussaian.