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Correction of the triplet skew quadrupole errors in RHIC

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1. Introduction

This study explores the possibility of operating the present RHIC coupling correction system in local decoupling mode, where a subset of skew quadrupoles are independently set by minimizing the coupling as locally measured by beam position monitors. The goal is to establish a correction procedure for the skew quadrupole errors in the interaction region triplets that does not rely on a priori knowledge of the individual errors.

After a description of the present coupling correction scheme envisioned for RHIC, the basics of the local decoupling method will be briefly recalled in the context of its implementation in the TEAPOT simulation code as well as operationally.

The method is then applied to the RHIC lattice: a series of simple tests establish that single triplet skew quadrupole errors can be corrected by local decoupling. More realistic correction schemes are then studied in order to correct distributed sources of skew quadrupole errors: the machine can be decoupled either by pure local decoupling or by a combination of global decoupling (minimum tune separation) and local decoupling technique. The different correction schemes are successively validated and evaluated by standard RHIC simulation runs with the complete set of errors and corrections.

The different solutions and results are finally discussed together with their implications for the hardware.

2. The present coupling correction system: hardware and strategy

The main sources of coupling in RHIC are systematic and random a_1 (skew quadrupole) multipoles in the dipoles and roll alignment errors in the quadrupoles. In particular, the triplet quadrupoles, strong and at a lattice position where the beta functions can be as large as 1300m, are a large source of coupling.

The coupling correction system for RHIC consists of 6 skew quadrupole families (8 quadrupoles in each family) located near the Interaction Regions (IRs) and 12 triplet correctors (1 skew quadrupole per triplet, embedded in the C2 corrector packages). It is worth noticing that the 6 families have in reality 12 independent power supply circuits, as described in [1]. There are further 36 skew quadrupoles distributed in the lattice but no power supplies are initially planned for them.

The correction scheme presently envisioned for RHIC relies on 4 families of skew quadrupoles set up to minimize the tune separation at the nominal operating tunes of 28.19 and 29.18. This condition is achieved by setting the deteminant of the matrix $h = m+n^+$ (where m and n are the off diagonal 2 by 2 matrices in the once around transfer matrix) to zero. This amounts to 2 independent conditions and in fact the 4 families are powered in antisymmetric pairs. A detailed description of this method can be found in [2]. The coupling effect of the triplets is corrected locally by the triplet skew quadrupole correctors by "dead reckoning" the triplet error, which is assumed known. Only the 4 triplet correctors in the low β^* IRs (6 and 8 o'clock) are

active. The triplet coupling correction is part of the general triplet correction scheme, which locally compensates for triplet multipole errors. Further details about the triplet correction system can be found in [3] and [4].

The local compensation of coupling caused by the triplets is necessary to achieve a good quality of correction. The motivation of investigating an alternative way to correct this effect arises from the fact the "dead reckoning" method works extremely well, provided we know the error. This may not always be the case: even if the triplet quadrupoles are carefully measured and aligned at the beginning, conditions may drift and cause uncorrected residual coupling errors. An operational way of removing the coupling caused by the triplets is desirable and will be discussed in the following.

3. Local decoupling: basics and operational implementation

The local decoupling technique is part of a general method for operational corrections of errors in accelerators. The general underlying concept is to determine the settings of correctors by minimization of a *"badness" function* that quantifies the effect to be corrected and that is built up by measurable quantities. The specific badness function will vary for the different correction operations that can be performed, like closed orbit correction, decoupling, correction of beta functions and vertical dispersion. A complete discussion of this general correction approach can be found in [5] and only those parts relative to the decoupling algorithm will be repeated here, that are necessary to explain the results obtained for RHIC. All the correction techniques are implemented in the TEAPOT simulation code [6] in an operational way that can be easily translated into application software procedures.

Coupled motion formalism.

Given the one turn 4 by 4 transfer matrix M,

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

in the cartesian space, it can be demonstrated that it is possible to find a coordinate transformation $x = G^T X$ to an eigenbasis where the 1-turn transfer matrix in the new coordinates is diagonal, i.e. has the form

$$\underline{M} = G^{T^{-1}} M G^T = \begin{bmatrix} \underline{A} & 0 \\ 0 & \underline{D} \end{bmatrix} \qquad \text{with} \qquad G^T = g \begin{bmatrix} I & R_D \\ R_A & I \end{bmatrix} \qquad \text{and}$$

$$R_A = \frac{C + \bar{B}}{\Lambda_A - trD} \qquad R_D = \frac{B + \bar{C}}{\Lambda_D - trD} \qquad g = \sqrt{\frac{|\Lambda_D - trA|}{|\Lambda_D - \Lambda_A|}}$$

where Λ_A and Λ_D are the eigenvalues of the matrix $M+\bar{M}$.

The $\underline{A}$ eigenmotion describes an ellipse in the (x,y) space and its major axis is tilted w.r. to the x axis by an angle θ_A given by:

$$\tan 2\theta_A = \frac{2R_{A11} - \left(\frac{\alpha_A}{\beta_A}\right) R_{A12}}{1 - \left[R_{A11} - \left(\frac{\alpha_A}{\beta_A}\right) R_{A12}\right]^2 - \left(\frac{R_{A12}}{\beta_A}\right)^2}$$

An analogous relation exists between the $\underline{D}$ eigenmotion and the y axis. The *eigenangles* θ_A and θ_D, not orthogonal in general, are a *measure of coupling* since for the ideal uncoupled case $\theta_A = \theta_D = 0$.

Another good measure of coupling is the area of the *eigenellipse*, given by $(\pi g^2/R_{A12})/\beta_A$ for the A eigen-

plane. If the coupling is weak, the areas of the 2 eigenellipses differ only by a multiplicative factor independent of coupling.

Measurable quantities

By driving the beam in such a way that only 1 mode is excited, the motion at one location in the lattice can be described in pseudoharmonic form:

$$x = g\cos\psi_A$$
$$y = g e_A \cos\left(\psi_A + \varepsilon_A\right)$$

where

$$e_A^2 = \left[R_{A11} - \left(\frac{\alpha_A}{\beta_A}\right) R_{A12}\right]^2 + \left(\frac{R_{A12}}{\beta_A}\right)^2$$

$$\varepsilon_A = -arc\tan \frac{R_{A12}/\beta_A}{R_{A11} - \left(\frac{\alpha_A}{\beta_A}\right) R_{A12}}$$

The x and y signals are coherent, being at the same frequency, hence their relationship at a specific position in the lattice is characterized by the ratio of amplitudes (e_A) and a phase difference (ε_A). By collecting turn by turn x and y positions at a double plane BPM, it is possible to measure the quantities e_A and ε_A with a network analyzer, and directly derive the matrix elements R_{A11} and R_{A12}. The coupling can be locally measured at every double plane BPM in the machine.

Badness and correction of coupling

In order to build a badness function for a minimization procedure, one needs a function that quantifies coupling and that goes to zero in the absence of coupling. As already discussed, the measurable quantities are e_A, which measures the ratio of out of plane vs. in plane oscillations and the phase difference ε_A.

A natural choice for the *coupling badness* B^C function is the following:

$$B^C = \sum_{d=1}^{N^d} e_A^2 \frac{\beta_x(d)}{\beta_y(d)}$$

The summation is taken over the number of detectors (BPMs) N^d and the ratio of β's assures that all detector measurements will have a comparable weight in the subsequent minimization process.

As already discussed, e_A is a function of the off diagonal matrix elements R_{A11} and R_{A12}, which in turn allow us to calculate the following influence functions:

$$R_{A11}(d) = R^o_{A11}(d) + \sum_{a=1}^{N^a} q_a^{skew} T_a^C(d)$$

$$R_{A12}(d) = R^o_{A12}(d) + \sum_{a=1}^{N^a} q_a^{skew} U_a^C(d)$$

The R^o functions represent the effect of the unknown errors at the position of detector d and the T^C and U^C functions can be calculated from the unperturbed lattice functions for every skew corrector a. By substitut-

ing the above expressions for R_{A11} and R_{A12} in the badness function B^C, the latter becomes a function of the N_a skew quadrupole corrector strengths q_a^{skew}. When $N^d > N^a$, as it is usually the case, one can determine the skew quadrupole corrector strengths by a fitting procedure so that the following conditions are met:

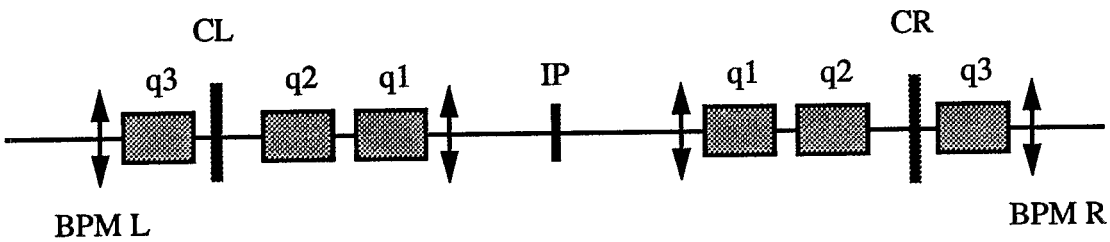
$$\frac{\partial}{\partial q_a^{skew}} B^C\left(q_1^{skew} \ldots\ldots q_{N_a}^{skew}\right) = 0 \qquad a = 1, \ldots N_a$$

The local coupling algorithm has been successfully applied to correct coupling in various lattices, the SSC Boosters and Collider as well as LEP. Experimental work towards the application of the method in existing machines has been carried out at HERA and LEP [7][8].

4. Application to RHIC: tests

Before studying and correcting more complex triplet skew quadrupole error distributions, the decoupling algorithm has been tested in one simple case when a *single roll error* is applied to one triplet quadrupole. A schematic view of the RHIC interaction region area is described in Figure 1 below. The arrows represent the beam position monitors and the foreseen triplet skew quadrupole correctors, labelled CL and CR, which are installed next to Q_3 triplet quadrupoles. (The triplet quadrupoles being at approximately the same betatron phase, only one skew quadrupole is needed to correct the effect of the whole triplet.)

Figure 1. Schematic view of the RHIC triplet region.



If we roll one of the triplet quadrupoles Q_j by an angle θ, the integrated skew multipole strength in the nearby corrector needed to compensate the error, can be calculated by:

$$a_1(corr) = \frac{-2\theta b_1\left(Q_j\right)\sqrt{\beta_x\left(Q_j\right)\beta_y\left(Q_j\right)}}{\sqrt{\beta_x(corr)\,\beta_y(corr)}}$$

In particular, if Q_3 is tilted by 1 mrad, the predicted setting for the nearby skew corrector is $-0.2119\ 10^{-3}\ m^{-1}$. The test consists in applying 1 mrad roll to the Q3 triplet quadrupole (and similarly to Q1 and Q2) in an otherwise ideal RHIC lattice, and checking the local coupling result versus the analytical one. The results for the

Q306 quadrupole are summarized in Table 1 and Figure 2a-d.

Table 1: Q3 triplet quadrupole rolled by 1 mrad in the 6 o'clock (1 m β*) interaction region.

CONFIGURATION	max eigenangle [degrees]	max vertical dispersion [m]	skew quadrupole corrector a_1 [m^{-1}]
no correction	45.0	~0.1	0
calculated setting	4.6	~0	$-0.2119\ 10^{-3}$
local decoupling (1 skew)	0.11	~0	$-0.2232\ 10^{-3}$
local decoupling (24 skew)	0.03	~0	$-0.2205\ 10^{-3}$

The first entry (and Figure 2a) describe the uncorrected effect of 1 mrad roll error in the Q_3 triplet quadrupole when the optics is tuned to β^*=1m (in the 6 o'clock and 8 o'clock IRs). The *minimum tune separation* in this case is 0.034. The second row (and Figure 2b) show the effect of dead reckoning the roll error by the calculated value, assuming the error known. As seen in the third row (and Figure 2c), the local decoupling algorithm can pinpoint the right correction setting when we use only the adjacent corrector strengths as a variable (1 skew case). If we activate other skew correctors distributed in the lattice (24 skew case), their strengths can be optimized to virtually suppress coupling far from the source in the machine (see Figure 2d). The corrector strengths of the remaining skew correctors are 2 orders of magnitude weaker than the corrector next to the Q_3 triplet. The same analysis has been repeated for the Q_2 and Q_1 triplet quadrupoles giving similar results.

The effect of *roll errors* (1 mrad) *distributed* in all the quadrupole in the lattice has also been checked for several random seeds. The uncorrected effect of the skew errors is summarized in Table 2 below. The *minimum tune separation*, here and in what follows, has been obtained 'experimentally' by asking the simulation code to tune the lattice to Q_x=28.185 and Q_y=29.185 and by calculating the difference in fractional tune after a fixed number of tuning attempts. (The minimization does not and is not expected to converge because the lattice is coupled). The nominal RHIC tunes are Q_x=28.19 and Q_y=29.18.

Table 2: Effect of a random distribution of roll errors in all quadrupoles.

seed	Q_x	Q_y	η_y^{max} [m]	max eigenangle [degrees]	ΔQ_{min}
0	28.20570	29.13903	0.10	45	0.06667
1	28.14583	29.22622	0.80	30	0.08039
2	28.20272 (28.19847)	29.14425 (29.15447)	0.70 0.45	45 45	0.05847 (0.04337)
3	28.24313	29.11197	0.20	45	0.13116
4	28.19209	29.17663	0.35	45	0.01216
5	28.19926	29.16786	0.20	45	0.03001

As it can be seen by comparing the result obtained for seed 2, where the value in parenthesis describe the effect of skew quad errors in the triplets only, the triplets are the dominant effect. Also, the effect depends noticeably on the error distribution, so for every scheme studied, the results have to be checked for a few random seeds.

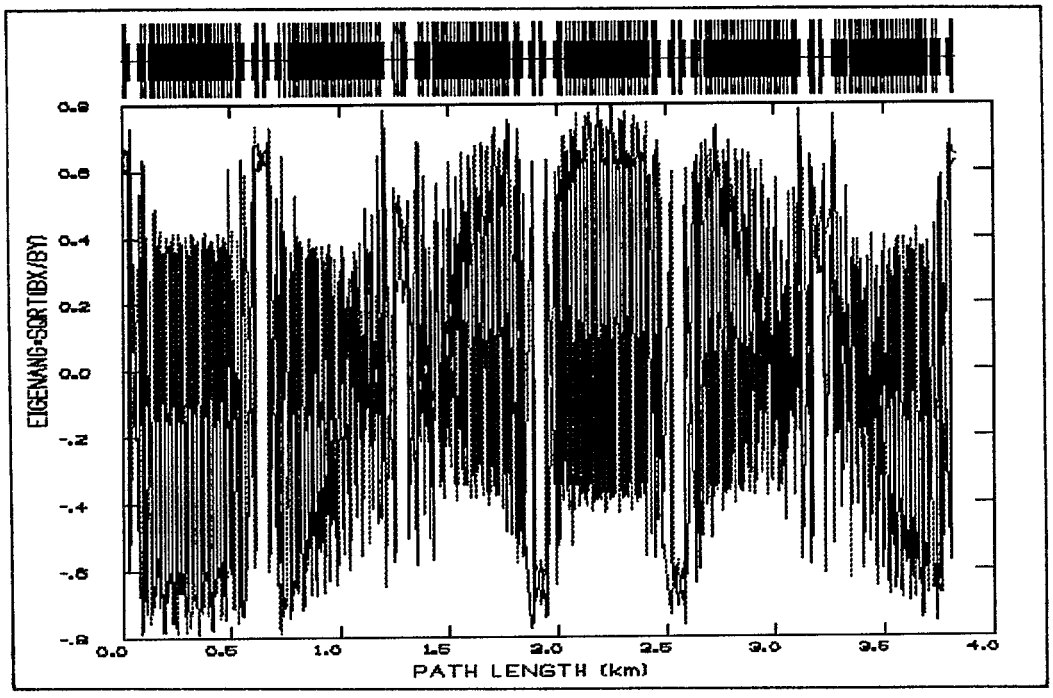


Figure 2a: 1 mrad roll error in Q_3, no correction: eigenangles[rad]

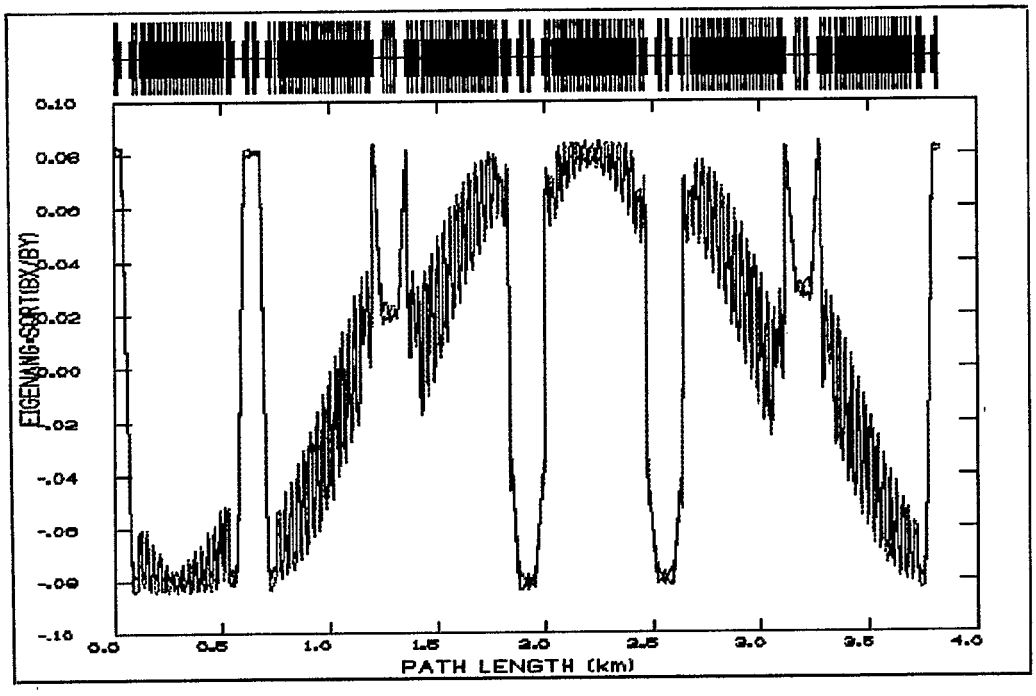


Figure 2b: Correction with calculated value (dead reckoning): eigenangles [rad]

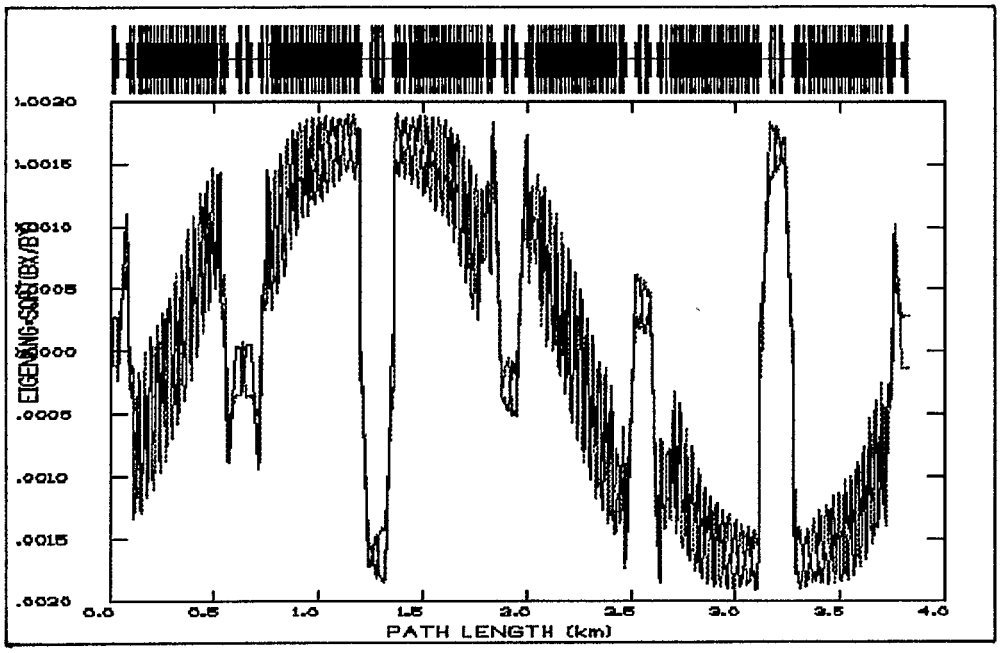


Figure 2c: Correction with local coupling algorithm (1 skew quadrupole corrector): eigenangles [rad]

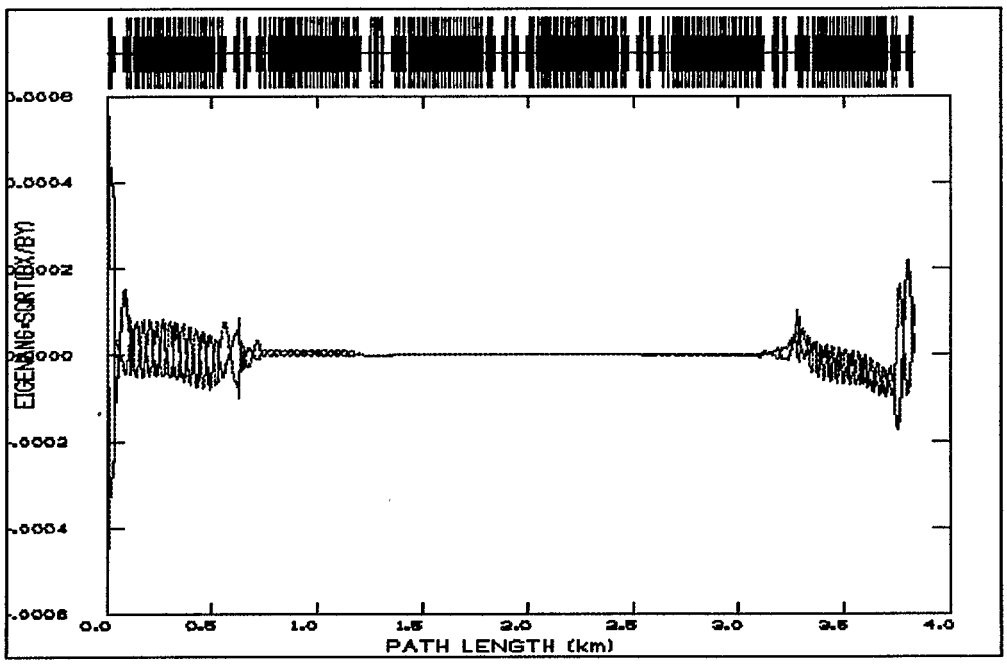


Figure 2d: Correction with local coupling algorithm (24 skew quadrupole correctors): eigenangles [rad]

5. Application to RHIC: local decoupling schemes

In order to study possible decoupling schemes, a random generation of skew quadrupole errors has been used in the lattice quadrupoles (triplet, IRs, arc) for the otherwise ideal RHIC lattice, in the storage configuration where 2 IRs (6 and 8 o'clock) are tuned to β^*=1m and the remaining 4 IRs to β^*=10m. At injection all the IRs are tuned to the higher β^* and hence the coupling caused by the triplets is lower.
As discussed in Section 3, local decoupling needs measurement of coupling and a number of independently powered skew quadrupole correctors. For the tests described and simulated here the assumptions are that we can measure coupling at every RHIC beam position monitor and that we have 12 independently powered skew quadrupole correctors located in the 12 RHIC triplets in the 6 IRs. The possibility of using 1 quadrupole out of every skew quadrupole family circuit has also been considered: that would provide a total of extra 12 skew quadrupoles that could be independently powered. (See Section 2). A more detailed discussion of the available and suggested hardware will follow in Section 8.
Several solutions of the following types have been investigated:

"pure" local solutions: *all skew errors* are corrected by local decoupling, with the 12 skew triplet correctors (local_12) or with 24 skew correctors (local_24: 12 triplet correctors and 12 from families circuits)

"hybrid" solutions: *arc-like errors* are corrected with 2 families set up to minimize the tune separation (global_2) and *triplet errors* are corrected with local decoupling (local_12 or local_4, where only the correctors in IR6 and IR8 are used)

Table 3 below summarizes the comparison of possible correction schemes for a random distribution of 1 mrad roll errors in all lattice quadrupoles. When all errors, triplet included, are corrected by global decoupling, the residual eigenangles and minimum tune separation are rather large. A typical *criterion for coupling correction* is to obtain eigenangles less than 10 degrees everywhere in the lattice. In the second and third case, arc-like errors and the high β^* triplet errors (in the 10, 12, 2, 4 o'clock IRs) are corrected by global decoupling and the low β^* triplet errors (6 and 8 o'clock IRs) are corrected by local decoupling, with 4 and 12 skew correctors respectively. The correction quality improves by using 12 correctors. The fourth case is global decoupling of arc-like quadrupoles and local correction of all triplet errors. The last two cases are 'pure' local decoupling corrections of all skew errors, with 12 and 24 skew quadrupoles respectively. In this case, all schemes other than the first give satisfactory results, especially when 12 triplet correctors are used to correct triplet errors.

Table 3: Comparison of coupling correction schemes for a (random) skew quadrupole error distribution

correction of arc-like errors	correction of high β^* triplet errors	correction of low β^* triplet errors	max eigenangle [degrees]	max vertical dispersion [m]	minimum tune separation
global_2	global_2	global_2	34.4	0.75	0.00758
global_2	global_2	local_4	5.7	0.60	0.00216
global_2	global_2	local_12	1.7	0.50	0.00002
global_2	local_12	local_12	1.7	0.40	0.00002
local_12	local_12	local_12	2.3	0.30	0.00090
local_24	local_24	local_24	0.08	0.36	0.00001

Tables 4 and 5 below summarize results for different random distributions of errors, for a *pure local decou-*

pling correction and the '*hybrid*' correction scheme.

Table 4: Correction of arc-like and triplet skew errors with 12 triplet skew correctors (local_12)

SEED	max eigenangle [degrees]	max vertical dispersion [m]	max skew quad kL [m^{-1}]
0	1.7	0.70	$0.813\ 10^{-3}$
1	1.1	0.36	$0.829\ 10^{-3}$
2	2.3	0.12	$0.217\ 10^{-2}$
3	1.0	0.40	$0.993\ 10^{-3}$
4	3.4	0.31	$0.483\ 10^{-3}$
5	1.7	0.37	$0.834\ 10^{-3}$

Table 5: "Hybrid" solution: correction of arc-like skew errors with 2 families (global_2) and triplet skew errors with local decoupling (local_12 or local_4)

SEED	triplet correction scheme	max eigenangle [degrees]	max vertical dispersion [m]	max skew quad kL [m^{-1}]
0	local_12 local_4	0.9 8.6	1.01 1.35	$0.948\ 10^{-3}$ $0.118\ 10^{-2}$
1	local_12	1.1	0.25	$0.610\ 10^{-3}$
2	local_12	3.4	0.34	$0.188\ 10^{-2}$
3	local_12 local_4	1.0 2.8	0.52 0.50	$0.974\ 10^{-3}$ $0.325\ 10^{-3}$
4	local_12 local_4	1.7 5.7	0.40 0.74	$0.817\ 10^{-3}$ $0.762\ 10^{-3}$
5	local_12	1.1	0.24	$0.873\ 10^{-3}$

A pure local decoupling solution as well as a hybrid solution with 12 triplet skew quadrupole correction look feasible on the basis of these results. The performance of the local decoupling with 12 independent skew quadrupole are better than with 4 skew quadrupoles.
Furthermore, for 3 error distributions, a satisfactory solution could not be directly found when only 4 skew quadrupoles were used (local_4), unless more manipulations, like applying the errors in 2 steps, were considered. Although that could be achieved, for instance by staged correction during the beta-squeeze process, it would add a further degree of complexity to the operation.
Figure 3a and 3b show the residual eigenangles and vertical dispersion for seed 3 when the local_12 scheme is used and Figure 4a and 4b show the same quantities for the local_4 scheme.

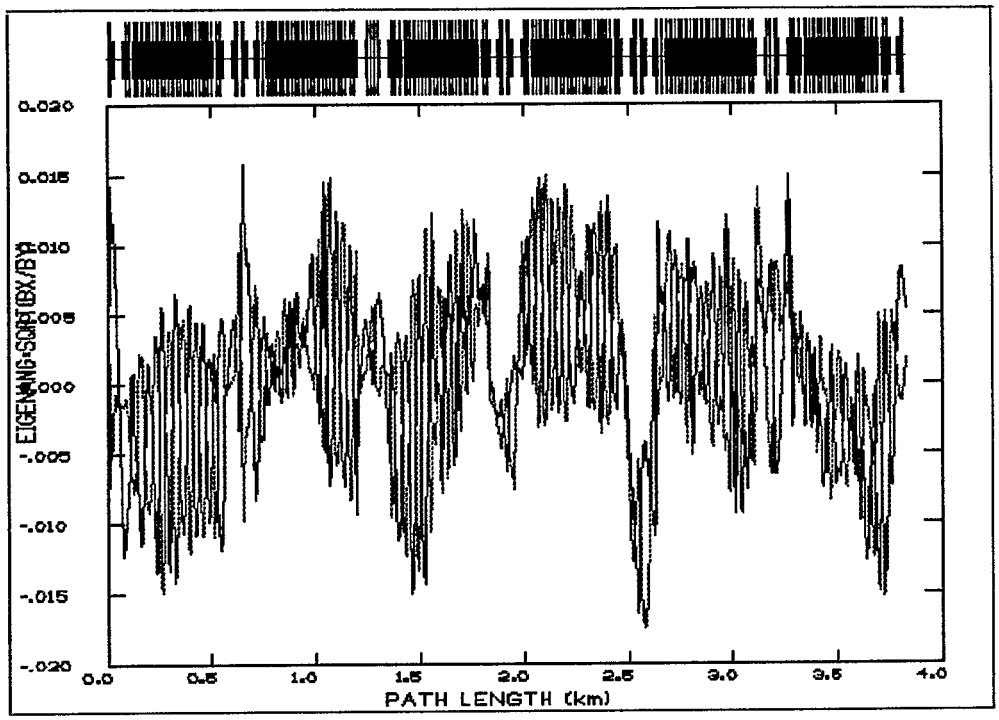


Figure 3a: Residual eigenangles [rad] for the hybrid solution (skew_12 scheme), seed 3

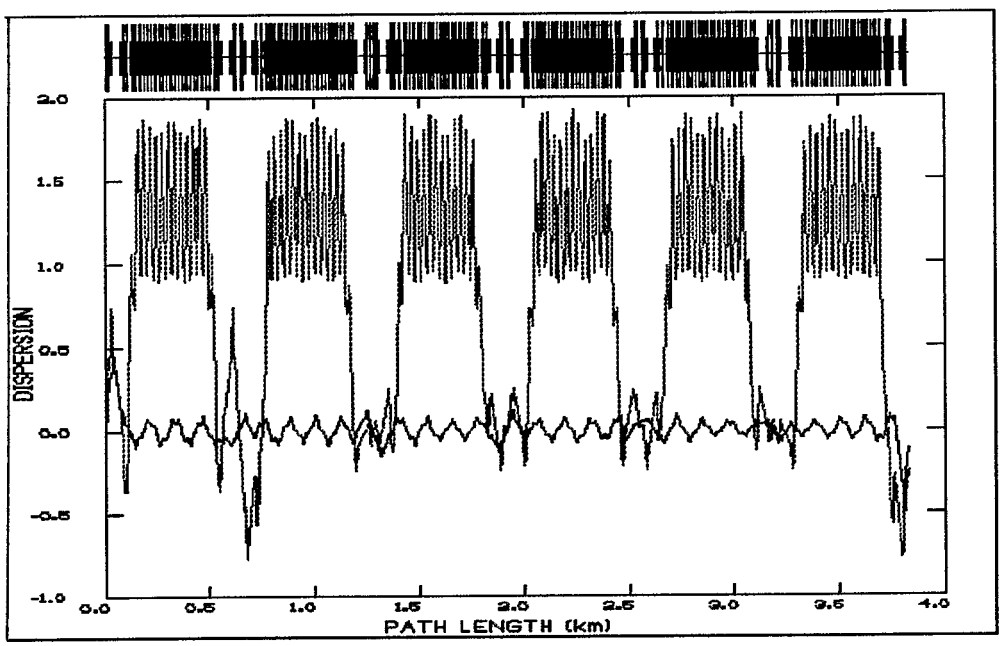


Figure 3b: Horizontal and residual vertical dispersion [m] for the hybrid solution (skew_12 scheme), seed 3

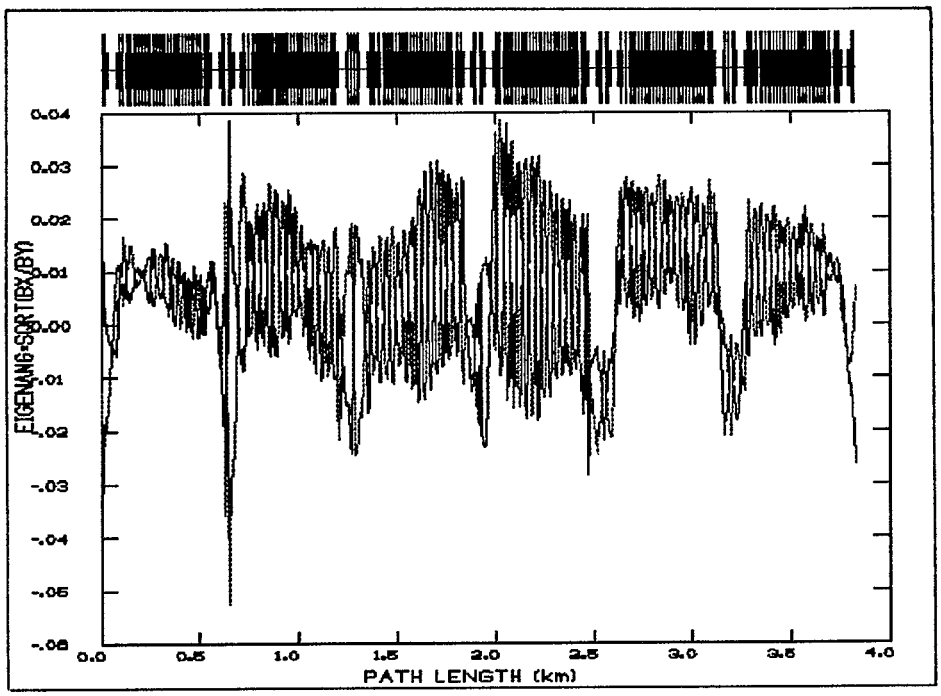


Figure 4a: Residual eigenangles [rad] for the hybrid solution (skew_4 scheme), seed 3

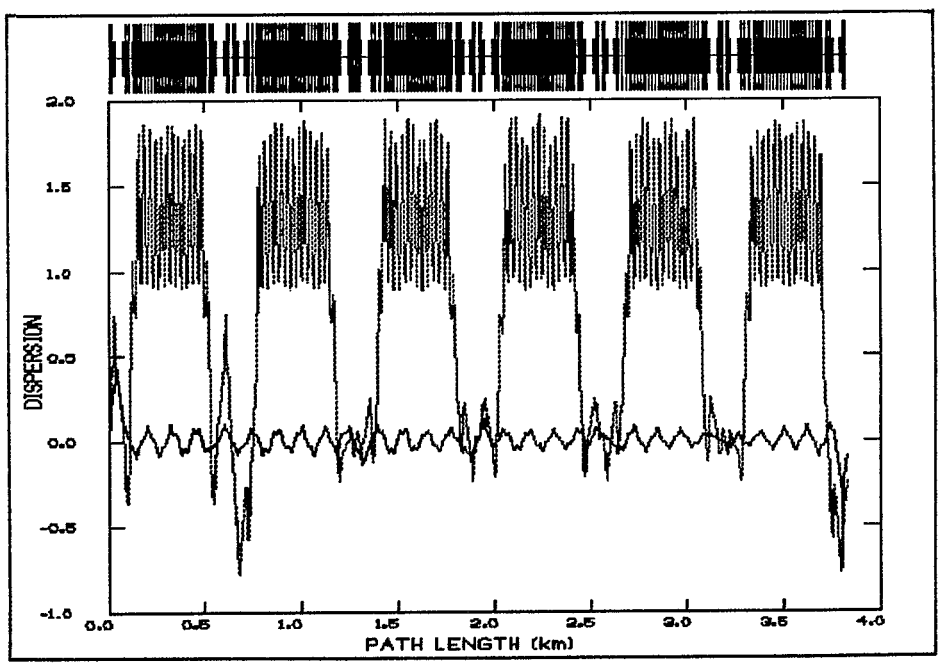


Figure 4b: Horizontal and residual vertical dispersion [m] for the hybrid solution (skew_4 scheme), seed 3

The maximum excitation allowed in the triplet C2 skew quadrupole correctors, 50 Amps corresponds to a maximum integrated strength of 1.46 10^{-3} m^{-1}. For the hybrid solution with 12 triplet skew quadrupoles operated in local decoupling mode, the quadrupole setting statistics over 6 seeds are:

mean: $< |kL| > = 0.420 * 10^{-3}\ m^{-1}$ or $< |I| > = 14.38$ Amps

sigma: $\sigma_{kL} = 0.295 * 10^{-3}\ m^{-1}$ or $\sigma_I = 10.10$ Amps

All the correctors are well within the system capability, with the exception of 1 skew corrector in seed 2, which exceeds the limit by 25%. In an instance like that, one can set the corrector to the maximum allowed strength and reapply the correction, with minimum impact on the correction quality.

6. Simulation with full set of errors

As already remarked, the systematic study of the triplet correction has been performed on an ideal RHIC storage lattice perturbed only by quadrupole roll error, for the conceptual reason of isolating the problem and the practical one of CPU time. The local coupling correction of the triplet errors has however been tested in the context of the full RHIC simulation, when all other errors and correction are also modelled.
The baseline 'MAC94.2' set of alignment and multipole errors as well as the RHIC standard set of corrections (tuning, chromaticity, triplet corrections) has been used, with the "dead reckon" compensation of triplet skew quadrupole errors substituted by local decoupling. The results for 4 error distributions are summarized in Table 6. Both the residual coupling and vertical dispersion are acceptable for RHIC and the skew quadrupole strengths required are in within the present system specifications.

Table 6: Simulation with the full set if errors and corrections

SEED	max eigenangle [degrees]	max vertical dispersion [m]	max skew quad $\|kL\|$ [m^{-1}]
0	3.1	0.34	0.771 10^{-3}
1	2.6	0.51	0.782 10^{-3}
2	3.7	0.62	0.778 10^{-3}
3	4.7	0.82	0.144 10^{-3}

Figure 5a and 5b show respectively the final result for the eigenangles and dispersions after all error and corrections are applied (seed 1).

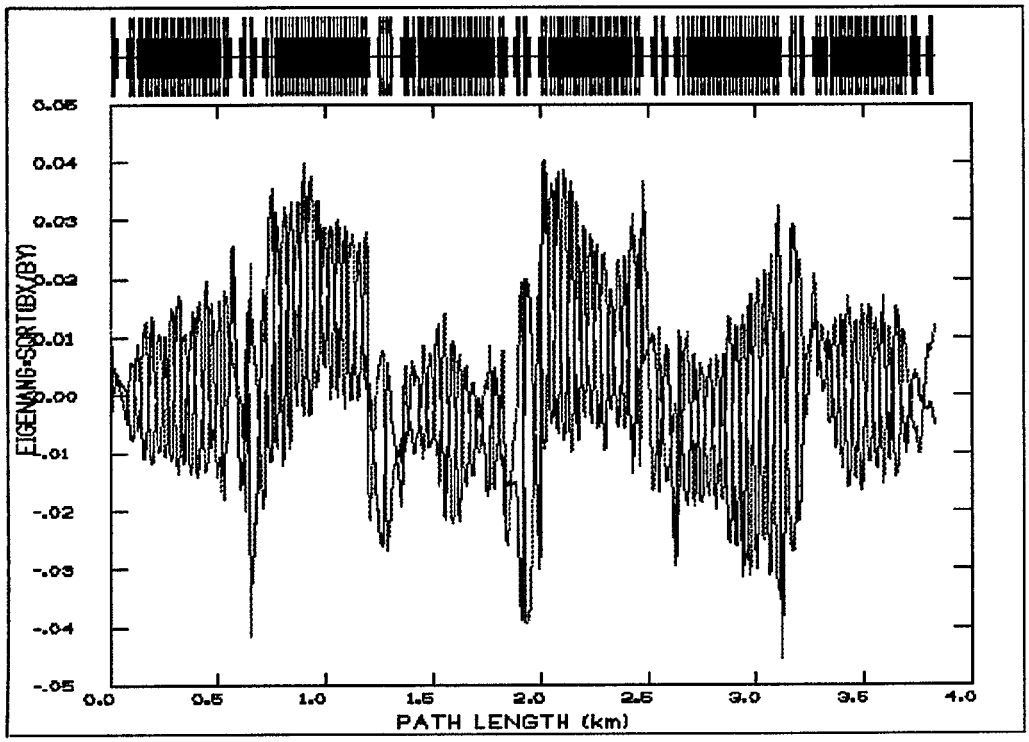


Figure 5a: Eigenangles [rad] in the presence of all errors and corrections

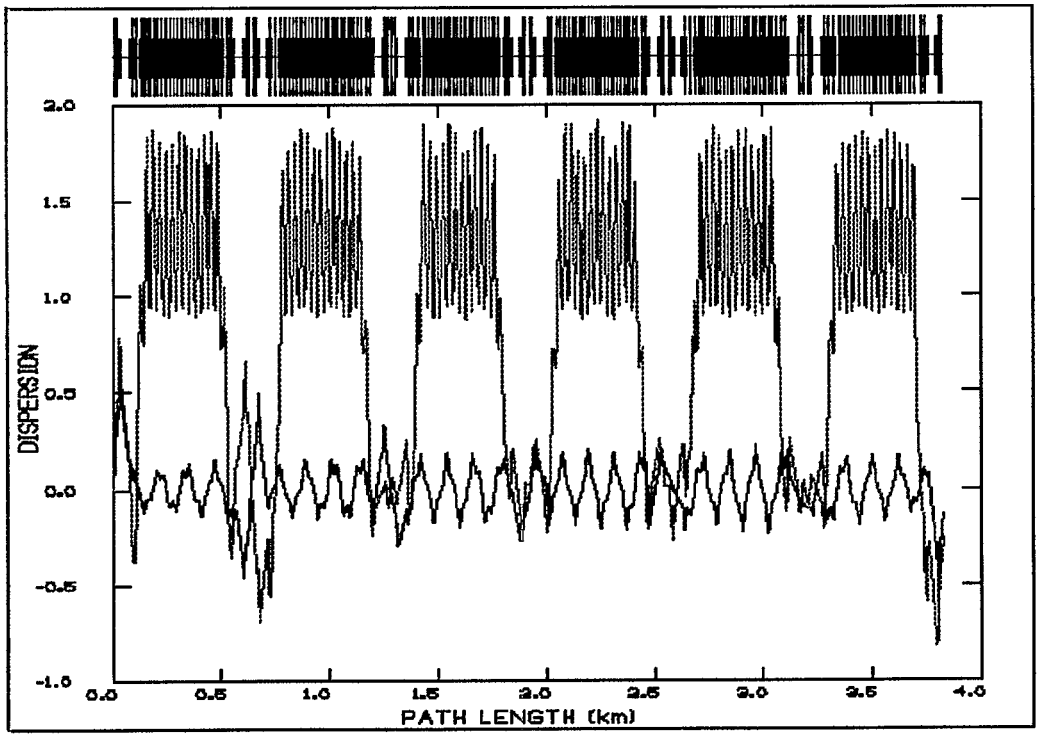


Figure 5b: Dispersions [m] in the presence of all errors and corrections

7. Discussion of results and their implication for the correction system.

The local decoupling technique proved effective in correcting triplet skew quadrupole errors by relying only on measurable quantities. The simulation results also showed that all coupling sources in the machine could be corrected by local decoupling, should that be desirable.

RHIC is adopting the solution to correct the skew quadrupole errors caused by the triplet with the 12 skew quadrupole correctors that are part of the C2 triplet corrector packages, and to rely on the minimum tune separation correction (2+2 skew quadrupole families) for correction of other coupling sources in the machine.
The old baseline corrector configuration for RHIC included power supplies only for the C2 correctors in the 6 and 8 o'clock interaction regions (low β^* triplets). The 12 skew scheme presented here would require 8 more power supplies for the high β^* C2 triplet correctors, a modest investment that will greatly improve the correction quality and reliability, as discussed in Section 5.

For the systematic study conducted here, the assumption was made that we can measure coupling at every beam position monitor (BPM) in the machine. Only a subset of RHIC BPMs are double view, the ones located in the interaction region areas, while the BPMs in the arcs are single plane. However, it was verified that the existing 2 plane BPMs provide enough coupling information for the preferred scheme (12 skew triplet correctors) to work.
In order to correct all coupling locally, coupling information from the arc is required: the local coupling algorithm implementation in TEAPOT is being extended so that the coupling at 1 arc horizontal (vertical) BPM can be inferred by measurements at the 2 adjacent vertical (horizontal) BPMs.

REFERENCES

[1] RHIC Design Manual

[2] D.Trbojevic, S.Tepikian, S.Peggs, *"Decoupling Correction System in RHIC"*, Proceedings of the "Orbit Correction and Analysis in Circular Accelerators" Workshop, BNL, December 1993

[3] J.Wei, R.Gupta, S.Peggs, *"Magnetic Correction of RHIC Triplets"*, Proceedings of the Particle Accelerator Conference (p. 258-260), Washington, May 1993

[4] S. Tepikian, S.Peggs, *"Skew Quadrupole Errors in the RHIC IR Triplets"*, Proceedings of the "Orbit Correction and Analysis in Circular Accelerators" Workshop, BNL, December 1993

[5] R.Talman, *"A Universal Algorithm for Accelerator Correction"*, Proceedings of the 'Advanced Beam Dynamics Workshop on effect of errors in accelerators, their diagnosis and correction", Corpus Christi, October 1991.

[6] L.Schachinger, R.Talman, *"Manual for the program TEAPOT"*, November 1994

[7] G.Bourianoff, *"Accelerator Simulation Activities at the SSCL"*, Proceedings of the "Stability of Particle Motion in Storage Rings" Workshop, BNL, October 1992

[8] G.Bourianoff, S.Hunt, D.Mathieson, F.Pilat, R.Talman, G.Morpurgo, *"Determination of coupled-lattice properties using turn-by-turn data"*, Proceedings of the "Stability of Particle Motion in Storage Rings" Workshop, BNL, October 1992

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