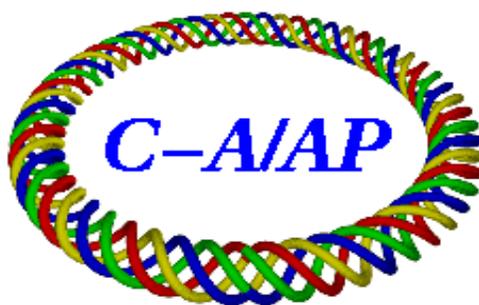


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An online application to measure the dispersion function in AGS

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Abstract

The knowledge of the experimentally measured dispersion function of the Alternating Gradient Synchrotron (AGS) is one of the requirements for the development of an accurate online model of the AGS. Currently the zgoubi and the MADX online models of the AGS are employed as online models to set up the beam optics of the synchrotron. In this technical note we describe the method we employ to measure the numerical values of the dispersion function of the AGS at the location of the Beam Position Monitors (BPM's) and we explain in details the automatic process to measure experimentally its dispersion function at the location of the BPM's. This automatic method of measuring the dispersion of the AGS not only reduces dramatically the time required to collect and analyze the experimental data to calculate the dispersion, but also minimizes the possible human error that may occur in the process of the dispersion measurement.

Definition of Dispersion

In this technical note we define as dispersion (D_x), the ratio:

$$D_x = \lim_{\substack{p \rightarrow p_0 \\ p_0}} \frac{\Delta x}{\left(\frac{p - p_0}{p_0}\right)} = \lim_{\Delta p \rightarrow 0} \frac{\Delta x}{\left(\frac{\Delta p}{p_0}\right)} = \left. \frac{\partial x}{\partial(\delta p)} \right|_{x=0} \quad (1) .$$
 In this definition of the dispersion, the symbol p_0

is the momentum of the central particle along the reference orbit, the symbol Δx is the transverse deviation from the reference orbit of an arbitrary particle having momentum p , and the symbol $\delta p = (p - p_0)/p_0$ is the relative momentum deviation from the reference particle p_0 . According to this definition if $D_x = 0$ then $\Delta x = 0$ thus the arbitrary particle having momentum (p) moves along the reference trajectory when $D_x = 0$. In a more rigorous but equivalent definition of the dispersion D_x , we can express the deviation Δx of the arbitrary particle with momentum p as a Taylor series expansion in terms of the relative momentum deviation $\delta p = (p - p_0)/p_0$ as in equation (2) below.

$$\Delta x(\delta p) = \left. \frac{\partial x}{\partial(\delta p)} \right|_{x=0} \delta p + \frac{1}{2!} \left. \frac{\partial^2 x}{\partial(\delta p)^2} \right|_{x=0} \delta p^2 + \frac{1}{3!} \left. \frac{\partial^3 x}{\partial(\delta p)^3} \right|_{x=0} \delta p^3 + \dots \quad (2)$$

The coefficient of the first term in the Taylor series expansion of the function $\Delta x(\delta p)$ as given by equation (2) is defined as the dispersion D_x .

Thus:

$$D_x = \frac{\Delta x}{\left(\frac{p - p_0}{p_0}\right)} = \lim_{\Delta p \rightarrow 0} \frac{\Delta x}{\left(\frac{\Delta p}{p_0}\right)} = \left. \frac{\partial x}{\partial(\delta p)} \right|_{x=0} \quad (3)$$

Based on the above definition of dispersion, given a pencil like beam where all particles with various momenta, move along a straight line along the beam direction ($D_x = 0$) before the pencil beam enters a

magnetic system, one can measure the dispersion of the beam at a particular point at the exit of the magnetic system, by measuring the transverse displacement Δx of a particles that has relative momentum difference $\delta = \Delta p/p_0$ from the central particle and subsequently forming the ratio $D_x = \Delta x/\delta$. To apply the definition of the dispersion as it is defined in this section we consider that the whole beam as it is circulating in the AGS consists of a single particle located at the center of charge of the beam which circulates on an average radius inside the AGS magnet.

Implementation of the Method

In this section we discuss the way we apply the definition of the dispersion as given in the previous section, to measure the dispersion of the AGS at the location of the BPM's. Most of the tasks of the procedure to measure the dispersion are automated.

1. To measure the dispersion of the AGS along a particular orbit of the circulating beam, we first have to place the beam orbit along this desired orbit. This can be accomplished by the use of the "AGS Orbit Control" application which allows us to place the centroid of the circulating beam, having a central (reference) momentum p_0 , on the desired "average beam radius" $\bar{r}_0(p_0, T0_j)$ by simply changing the values of the "radial function". This is equivalent of placing the reference momentum p_0 of the beam along the desired reference orbit which has an "average beam radius" $\bar{r}_0(p_0, T0_j)$. The symbol $T0_j$ is the time during the acceleration cycle we want to measure the dispersion of the AGS on the "average beam radius" $\bar{r}_0(p_0, T0_j)$. The index j refers to the number of $T0$ times the orbit is measured during the acceleration cycle (magnet cycle).

2. We open the "AGS Orbit Display" application and we set the application to measure the position of the centroid of the orbit $x_{cent}(p_0, T0_j, BPM_i)$ at the location of the Beam Position Monitors (BPM's), by specifying the times $T0_j$ during the magnet cycle. From these positions $x_{cent}(p_0, T0_j, BPM_i)$ the application calculates the average radius $\bar{r}_0(p_0, T0_j)$ of the beam orbit at the various selected $T0_j$ times during the magnet cycle. This "average radius" $\bar{r}_0(p_0, T0_j)$ is calculated from the position $x_{cent}(p_0, T0_j, BPM_i)$ of the beam's centroid at the location of the BPM's by the

use of relation:
$$\bar{r}_0(p_0, T0_j) = \frac{\sum_{k=1}^{n_{BPM}} x_{cent}(p_0, T0_j, BPM_k)}{n_{BPM}}$$
 The variable n_{BPM} is the number of the BPM's along the AGS ring where the position of the beam has been measured.

3. With the tasks 1 and 2 we have measured the position $x_{cent}(p_0, T0_j, BPM_k)$ of the reference orbit p_0 at the location of the BPM's and we have calculated the average radius $\bar{r}(p_0, T0_j)$ of the circulating beam at various times ($T0_j$) during the acceleration cycle. Now we must measure the position of the beam's centroid $x_{cent}(p_i, T0_j, BPM_i)$ at the location of the BPM's of an arbitrary momentum (p_i) and subsequently to calculate the average radius $\bar{r}(p_i, T0_j)$ of the arbitrary

momentum (p_i). We therefore repeat tasks 1 and 2 above as they are described in tasks 4 and 5 below.

4. We modify the radial function in the “AGS Orbit Control” application (this is done automatically by the application) to change the average momentum of the beam bunch to the prescribed value of the arbitrary momentum (p_i).

5. The new average momentum (p_i) will move the centroid of the beam to a new average orbit $\bar{r}_i(p_i, T0_j)$ and the application will provide the location $x_{cent}(p_i, T0_j, BPM_k)$ of the centroid of the beam at the location of the BPM's. This average radius $\bar{r}_i(p_i, T0_j)$ of the arbitrary momentum (p_i) is calculated by the “AGS Orbit Display” application using again the formula

$$\bar{r}_i(p_i, T0_j) = \frac{\sum_{k=1}^{n_{BPM}} x_{cent}(p_i, T0_j, BPM_k)}{n_{BPM}} \quad \text{Although in practice these two sets of measurements (tasks 3}$$

and 5) are sufficient to determine the value of the dispersion function at the location of the BPM's by using the definition of dispersion in equation (3), we execute step 6 below to calculate the values of the dispersion function with better accuracy.

6. In this task we repeat steps 4 and 5 few more times but each time with a new “arbitrary value of momentum p_i ”. This provides a set of position values at the location of the BPM's as a function of “arbitrary beam momentum p_i ” or “average radius” $\bar{r}_i(p_i, T0_j)$ of the circulating beam. This is equivalent to generating a function $\Delta x(\delta p_i, T0_j, BPM_k) = x_{cent}(p_i, T0_j, BPM_k) - x_{cent}(p_0, T0_j, BPM_k)$ with an independent variable of δp_i . Now the discrete values of $\Delta x(\delta p_i, T0_j, BPM_k)$ can be fitted to a polynomial (we have chosen a 2nd order polynomial) which can be expanded about a particular p_i (we usually choose $p_i = p_0$) as a Taylor series in terms of δp_i , (see equation 2 above), to provide the value of the dispersion at the location of each BPM.

7. To further increase the accuracy of the measured dispersion we measure the centroid of the beam five times (five AGS cycles) for a given beam momentum p_i and we calculate an average centroid of the beam $\bar{x}_{cent}(p_i, T0_j, BPM_k)$ at the location of each BPM using the formula:

$$\bar{x}_{cent}(p_i, T0_j, BPM_k) = \frac{\sum_{n=1}^{AGS_cycles} x(n, p_i, T0_j, BPM_k)}{AGS_cycles} \quad \text{Subsequently we form the function}$$

$\Delta \bar{x}_{cent}(\delta p_i, T0_j, BPM_k) = \bar{x}_{cent}(\delta p_i, T0_j, BPM_k) - \bar{x}_{cent}(p_0, T0_j, BPM_k)$ which can be expanded as a Taylor series in terms of δp_i , as in equation 2 above, to provide the value of the dispersion at the location of the BPM's.

Details in setting up the automatic Dispersion measurement

In this section we describe in greater detail the procedure to measure the dispersion of the AGS at the location of the BPM's. The Application which is used to measure the dispersion of AGS comes under the name: "AgsDispersion" and its source-fortran-code "AgsDispersion.f" is located in the directory:

/operations/app_store/AgsDispersionMeasure/dispersionCalculation/AgsDispersion/

The automatic dispersion measurement procedure can be separated in two parts;

- a) the data collection-part, and
- b) the analysis of the data

Although this separation in two parts is transparent to the user, we describe below each part separately to better clarify the procedure and provide explanation on the method we use to calculate the dispersion.

Before the execution of the program "AgsDispersion" which collects and also does the analysis the data, we should open the "AGS Orbit Display" application to enter the following quantities:

1. Set the active PPM User.
2. Set the times $T0_j$, during each magnet cycle, at which we want to measure the dispersion.
3. Set the number of orbits for each particular beam radius (usually 5). The measurement of a single orbit requires a single magnet cycle. By increasing the number of the measured orbits (number of AGS magnet cycles per orbit) we increase the accuracy in the dispersion measurements, and also we allow for magnet cycles without beam to be discarded just in case the beam fails to be injected into the AGS during a magnet cycle. Magnet cycles without beam are discarded and are not included in the calculations.
4. We run the "AGS Orbit Display" application to measure the "average beam radius" $\bar{r}_0(p_0, T0_j)$ at the various $T0_j$ times along the magnet cycle. If we are satisfied with the measured values of the "average beam radii" $\bar{r}_0(p_0, T0_j)$ we proceed to the following section, otherwise we use the "AGS Orbit Control" application to change the function to obtain the required values of the "average beam radii" $\bar{r}_0(p_0, T0_j)$ we want to measure the dispersion.

Running the code

Having set up the “AGS Orbit Display” application (steps 1 to 4 above) we may run the automatic dispersion measurement code.

We have two options to run the application:

- a) Run the application using a window while we are in a directory of our choice.
- b) Run the application from the StartUp menu.

Running the code from a window

To run the code from a window we should make a directory from which we must execute the code. All the results and plots regarding the dispersion calculations will be written in this directory.

Below we are describing the steps required to run the code.

- a) From the directory we created we execute the following line:
`/operations/app_store/AgsDispersionMeasure/dispersionCalculation/AgsDispersion/AgsDispersion -inFile`
- b) If in the directory we executed the code in step (a) above there is no file under the name “AgsDispersion.in” (see explanation below) the code will print on the window the following lines:
Please enter in one line below:
step (Volt), number-of-steps, t1 (ms), t2 (ms), user#, debug (0/1), disp-plot(0/1), cfact (~1.8)

step (Volt) : This is the numerical value of the step which will change the value of the radial steering function in the time range **t1 (ms), t2 (ms)**, defined below. This quantity is in volts and corresponds to the beam’s momentum-step δp required to place the beam into a new orbit.

number-of-steps : Number of momentum steps δp in one direction. This argument corresponds to the number of orbits. As an example; if **number-of-steps=1** the number of orbits will be three ($-\delta p, 0, \delta p$) if **number-of-steps=2** the number of orbits will be five ($-2\delta p, -\delta p, 0, \delta p, 2\delta p$)

t1 (ms), t2 (ms) : Start/Finish “T0” times to apply the **step** to the radial steering function. In this time interval t1-t2 the beam momentum will change by the value defined by the values of **step** and **number-of-steps** therefore the beam orbit will be shifted accordingly in the time interval **t1 (ms), t2 (ms)**.

user# : Active PPM User. The number of the active PPM User (1 through 4)

debug (0/1) : A flag for debugging; usually set 0

disp-plot(0/1) : A flag for plotting the dispersion function at the end of the analysis. Usually set **1**

cfact : This value converts the calculated quantity Δx_{cod} (mm)/AvRad(mm) to $[m/(\delta p/p)]$ (units of dispersion). The value of the “**cfact**” can be changed if we reanalyze the measured orbits. (See section below “reanalyzing the data”)

After entering the above values the computer code asks to enter the path of the directory where the results from the analysis of the experimental data will be written.

Please enter output directory

In answer to this question, we usually enter the symbol “.” as a path directory. This symbol means the “current directory” in which we execute the code, and in this directory will be written the results of the analysis as well as the final results and plots of the dispersion.

At this stage of the application the code prints on the window the data we have entered previously, as shown below, and also outputs this data on a file “**AgsDispersion.in**” which resides on the directory we executed the code.

The four lines below is the output of the computer code on the window we executed the code.

Here are the settings :

```
1.0000E+00 1 2000 2200 4 0 1 1.8000 step, #of-steps, start-time, stop-time, ppmuser, debug, disp-plot, cfactor
```

- This dot is part of the settings and signifies the “current directory”.

Do you want to change it ? (N/Y)

If we answer with “N” or “n” to the above question, the computer code continues to collect the experimental and calculate the dispersion.

If we answer with “Y” or “y”, because we made a mistake in entering the correct input data and we want to correct them, the computer code stops and prompts the user with the statement below:

emacs file AgsDispersion.in, and change these settings in there. Then run again.

After we edit the file **AgsDispersion.in** we run again by executing the statement in item (a) above. This time the code prints on the window the following four line which will have the desired settings:

Here are the settings :

```
1.0000E+00 1 2000 2200 4 0 1 1.8000 step, #of-steps, start-time, stop-time, ppmuser, debug, disp-plot, cfactor
```

- This dot is part of the settings and signifies the “current directory”.

Do you want to change it? (N/Y)

Now assuming that the input data is correct we answer with “N” or “n” and the computer code continues to collect the experimental data and calculate the dispersion.

As the next step, which is transparent to the user, the computer executes the script below.

/operations/app_store/AgsDispersionMeasure/bin/AgsDispersionMeasure.csh

The function of the script above is to establish communication between the following applications:

1. The “AGS Orbit Control” application which controls the beam orbit in the AGS.
2. The “AGS Orbit Display” application which measures the position of the orbit at the location of the BPM’s.

An additional function of the script “AgsDispersionMeasure.csh” is to write the measured orbits as collected from the “AGS Orbit Display” application on a directory which has the generic name: “**user#.t1tot2ms.time-stamp**”

The path of this directory is given below:

```
/operations/app_store/AgsDispersionMeasure/data/user#.t1tot2ms.time-stamp
```

The directory “**user#.t1tot2ms.time-stamp**” in the path above, is created each time the script “AgsDispersionMeasure.csh” is executed. The name of the directory is self-explanatory because the name of the directory is related to the input data as explained above. The phrase “time-stamp” in the directory is the date and the time we collect the data. Additional explanation about the meaning of the filename is provided below.

In this directory “**user#.t1tot2ms.time-stamp**” the filenames of the orbits collected by the “AGS Orbit Display” application are written.

After the orbit data is collected and written in the directory “**user#.t1tot2ms.time-stamp**” the executable code “AgsDispersion” reads the filenames of the orbits from the directory “**user#.t1tot2ms.time-stamp**” and writes them on a file “dispersionFrom_AgsOrbitDisplay.in” . This file is created at the directory we execute the code “AgsDispersion” and its content is shown below. An example of an actual directory name “**user#.t1tot2ms.time-stamp**” is: U4.170to220ms.2013-04-15-12.57.03. This means that the application is set in user#4 and the radial average change is set in the time interval 170 ms to 220 ms. The data was collected at date and time of 2013-04-15-12.57.03.

The content of the file “dispersionFrom_AgsOrbitDisplay.in” is:

```
5 ! Number of Orbit files measured by the “AGS Orbit Display” application; The path and file names of the measured orbits are written below.  
0.0 ! The dispersion will be calculated at the average radius r=-0.0 mm with respect to the radius of the reference orbit which is 0 mm  
1.8 ! Conversion factor to convert the Average_Beam_Radius[mm] to [dp/p]  
  
/operations/app_store/AgsDispersionMeasure/data/U4.170to220ms.2013-04-15-12.57.03/BEFOREDISPMEAS.U4.2013-04-15-12.57.03  
/operations/app_store/AgsDispersionMeasure/data/U4.170to220ms.2013-04-15-12.57.03/DM.U4.NEG2.S1.2013-04-15-12.57.03  
/operations/app_store/AgsDispersionMeasure/data/U4.170to220ms.2013-04-15-12.57.03/DM.U4.NEG2.S2.2013-04-15-12.57.03  
/operations/app_store/AgsDispersionMeasure/data/U4.170to220ms.2013-04-15-12.57.03/DM.U4.POS2.S1.2013-04-15-12.57.03  
/operations/app_store/AgsDispersionMeasure/data/U4.170to220ms.2013-04-15-12.57.03/DM.U4.POS2.S2.2013-04-15-12.57.03
```

What follows is an explanation of each line appearing above in the file “dispersionFrom_AgsOrbitDisplay.in”

Line 1: (Usually set equal to 5) is the number of orbit files requested and measured by the “AGS Orbit Display” application

Line 2: (Usually set equal to 0.0) Indicates the value of the average radius (0.0 mm) that the Taylor series expansion will be performed by the code to calculate the dispersion at this average radius. If we want the

value of the dispersion at a different average radius, like -1.0 mm for example, we may reanalyze the data. (See section below)

Line 3: (Usually set equal to 1.8) Is a conversion factor converting the change of “average radius” ΔR in mm to relative momentum (δ).

Lines 4 through 8: are the filenames of the orbits collected at various average radii. In this case the first file (2nd line) contains the data from the starting orbit and it is usually considered as the reference orbit. The “NEG2” indicates that 2 orbits with radii $-1*\text{step}=\text{“S1”}$ and $-2*\text{step}=\text{“S2”}$ have been collected in the negative direction and the “POS2” indicates that 2 orbits with radii $+1*\text{step}=\text{“S1”}$ and $+2*\text{step}=\text{“S2”}$ have been collected in the positive direction.

Running the code from the StartUp

The program can be executed as an application which resides in StartUp. All the required arguments are provided in a window of the application shown in Figure 1 below. The meaning of the values in the boxes shown in the window of the application (see Figure 1) are identical to the values of the values of the quantities we enter when we run the code from a window and are explained in a previous section under the title ***“Running the code from a window”*** However we repeat their explanation below again.

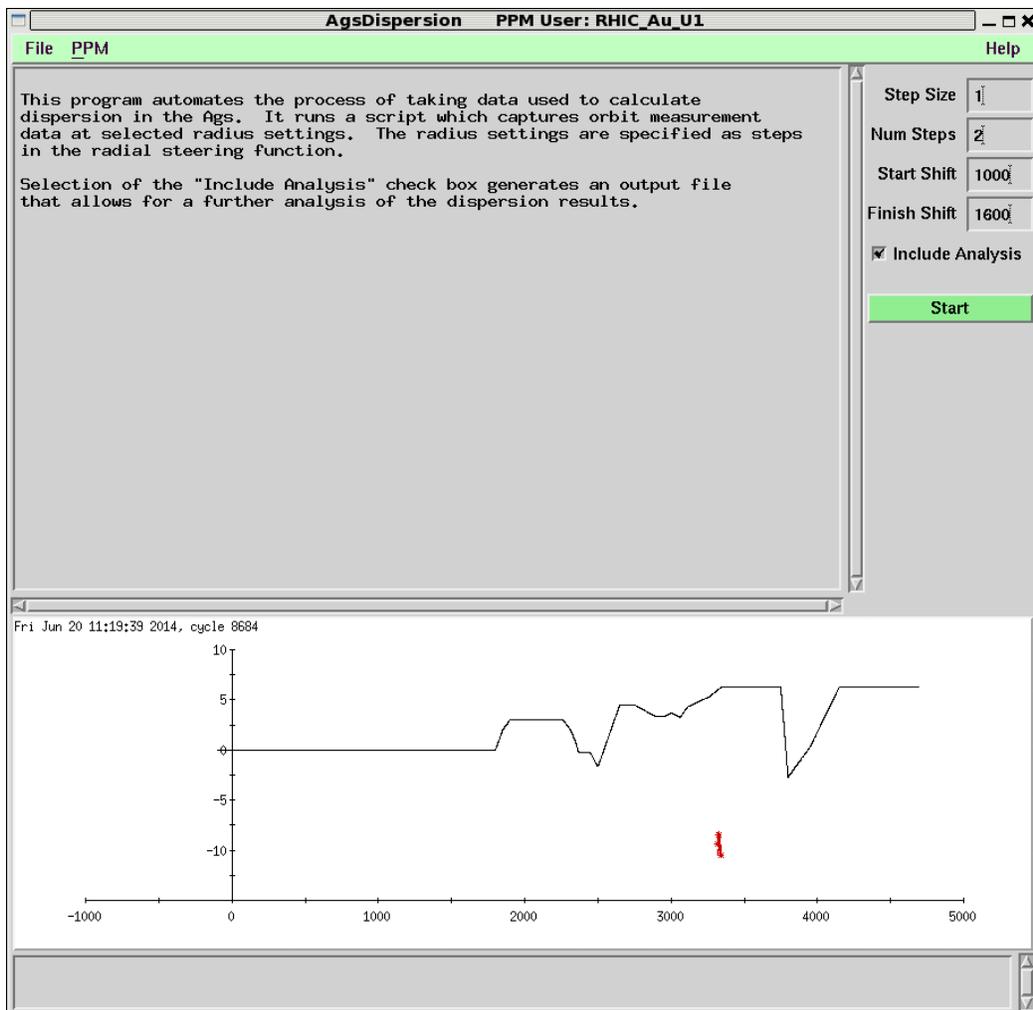


Figure 1 The application window appearing when we run the application from the StartUp.

Step Size (Volt) : This is the numerical value of the step which will change the radial steering function in the time range **t1 (ms)**, **t2 (ms)**, defined below. This quantity is in volts and corresponds to the beam’s momentum-step δp required to place the beam into a new orbit.

Num Steps=number-of-steps : Number of momentum steps δp in one direction. This argument corresponds to the number of orbits. As an example; if **number-of-steps=1** the number of orbits will be three $(-\delta p, 0, \delta p)$ if **number-of-steps=2** the number of orbits will be five $(-2\delta p, -\delta p, 0, \delta p, 2\delta p)$

Start Shift=t1 (ms), Finish Shift=t2 (ms) : Start/Finish “T0” Time to apply the **step** to the radial steering function. In this time interval $t1-t2$ the beam momentum will change by the value defined by the values of **step** and **number-of-steps** therefore the beam orbit will be shifted accordingly in the time interval **t1 (ms)**, **t2 (ms)**.

The **user#** : Active PPM User. The number of the active PPM User (1 through 4) can be entered by left-clicking in the “PPM” button appearing in the top-left side of the application window shown in Figure 1.

The box next to the phrase “Include Analysis” should be checked if we want the collected data (orbits) to be analyzed and provide the dispersion function values at the location of the BPM’s.

By clicking on the **Start** button of the application window (see Figure 1) the window shown in Figure 2 appears. In this window the user can select the directory in which the results from the data analysis will be written.

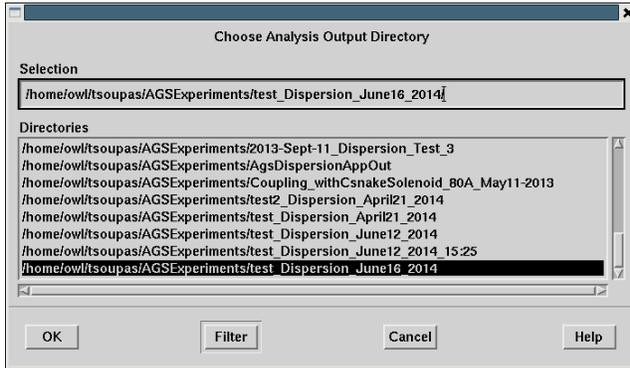


Figure 2. The application window in which the user can select the directory in which the results from the analysis of the data will be written.

After selecting the directory in which the results from the data analysis are recorded, we click the Button “OK” (see Fig. 2) and the window shown in Figure 3 appears on the screen. This is the last window of the application before the data collection and analysis is shown in Figure 3. This window contains important information for the user who is new with the application. After reading the content of the window the user should click OK and the application will proceed with data collection and analysis. In the subsection below we describe in some details the process of the data analysis for the calculation of the dispersion.

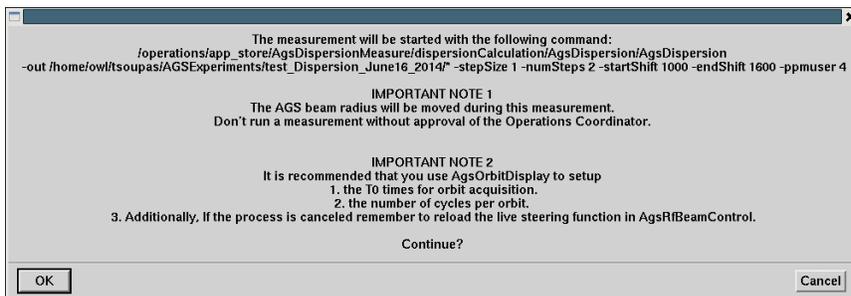


Figure 3. This is the last window of the application before the data collection and analysis. It only contains important information for the user who is new with the application.

The data analysis and calculation of the dispersion

In this section we provide information on the part of the computer code which calculates the dispersion from the experimental data which is collected by the data collection part of the code discussed earlier. This part of the application is transparent to the user but we include it to provide more complete information on how the dispersion is calculated from the experimental data.

The data-analysis fortran computer code “dispersionFrom_AgsOrbitDisplay.f” and its executable version “dispersionFrom_AgsOrbitDisplay” which calculate the dispersion are located in the directory:

/operations/app_store/AgsDispersionMeasure/dispersionCalculation/source/.

Upon execution of the computer code “dispersionFrom_AgsOrbitDisplay” the code reads data from the input data file “dispersionFrom_AgsOrbitDisplay.in”. This input data file has been generated by the “data collection part” of the code and the information contained in this file is provided in the previous section. The data of each orbit file listed in the data-input-file “dispersionFrom_AgsOrbitDisplay.in” contains the following information:

1. The “number of the AGS cycles” which were used to collect each orbit with a particular average radius.
2. The “number of T_0 times” the beam position at the BPM’s was measured for this particular average radius.
3. The readings of the 72 BPM’s (horizontal beam orbits) which correspond to the horizontal position of the beam’s centroid. The number of sets of the 72 BPM’s readings is equal to the “number of the T_0 times” (item 2 above) we have requested the “AGS Orbit Display” application to collect.
4. The readings of the 72 BPM’s (vertical beam orbits) which correspond to the vertical position of the beam’s centroid. The number of sets is equal to the “number of the T_0 times”.
5. The readings of the 72 BPM’s which correspond to the beam current as measured by the BPM’s. The number of sets is equal to the “number of the T_0 times”.

The code reads these data from each orbit file to make sure that the data is valid. Valid data is considered when the beam current as measured by the BPM’s (item 5 above) is above a give value. In case the beam current is below this prescribe value, a “bad-orbit-flag” is raised for each “bad orbits”. The orbits which are assigned the “bad orbit” flag are excluded from the calculations.

Subsequently the code reads again the orbit files which contain “good data” to calculate the location of the average beam centroid $\bar{x}_{cent}(p_i, T0_j, BPM_k)$ at the location of each BPM_i , for each beam momentum p_i , and time $T0_j$. The orbits flagged as “bad orbits” are excluded from the calculations of the dispersion.

The average position of the beam centroid at the location of the BPM_k is given by the relation 4 below:

$$\bar{x}_{cent}(p_i, T0_j, BPM_k) = \frac{\sum_{n=1}^{AGS_cycles} x(n, p_i, T0_j, BPM_k)}{AGS_cycles} \quad (4) \quad \text{In equation (4) } p_i \text{ is the central momentum of the}$$

beam and corresponds to a given “average beam radius” of the circulating beam. This “average beam

radius” is calculated by the formula

$$\bar{r}_i(p_i, T0_j) = \frac{\sum_{k=1}^{n_{BPM}} \bar{x}_{cent}(p_i, T0_j, BPM_k)}{n_{BPM}} \quad (5)$$

One of these orbit files is the “reference file” and contains the position of the orbit at the BPM’s. The average position of the beam centroid $\bar{x}_{cent}(p_0, T0_j, BPM_k)$ of the reference orbit $\bar{r}_0(p_0, T0_j)$ at the

location of the BPM’s is calculated by:
$$\bar{x}_{cent}(p_0, T0_j, BPM_k) = \frac{\sum_{n=1}^{AGS_cycles} x(n, p_0, T0_j, BPM_k)}{AGS_cycles} \quad (6)$$

In equation (6) p_0 has the same meaning as p_i and corresponds to the “average beam radius” of the circulating beam along which we want to calculate the dispersion of the AGS. This “average beam

radius” of the reference orbit is given by:
$$\bar{r}_0(p_0, T0_j) = \frac{\sum_{k=1}^{n_{BPM}} \bar{x}_{cent}(p_0, T0_j, BPM_k)}{n_{BPM}} \quad (7)$$

In the expressions (4,5,6,7) the symbol BPM_k , corresponds to any of the 72 BPM’s of the AGS which provides valid beam position readings. The symbol n_{BPM} in expressions (5, and 7) is the total number of the BPM’s which provide valid data for the beam position.

Thus each beam central momentum p_i is characterized by:

a) An average location at each BPM,
$$\bar{x}_{cent}(p_i, T0_j, BPM_k) = \frac{\sum_{n=1}^{AGS_cycles} x(n, p_i, T0_j, BPM_k)}{AGS_cycles} \quad (8)$$

b) An “average beam radius” calculate as:
$$\bar{r}_i(p_i, T0_j) = \frac{\sum_{k=1}^{n_{BPM}} \bar{x}_{cent}(p_i, T0_j, BPM_k)}{n_{BPM}} \quad (9)$$

c) One of these orbits is chosen as “reference orbit”
$$\bar{r}_0(p_0, T0_j) = \frac{\sum_{k=1}^{n_{BPM}} \bar{x}_{cent}(p_0, T0_j, BPM_k)}{n_{BPM}} \quad (10)$$
 along

which the dispersion of the AGS will be calculated.

At the location of each BPM the code calculates pairs $\{\bar{x}_{cent}(p_i, T0_j, BPM_k), \bar{r}_i(p_i, T0_j)\}$ of the “average beam position at the BPM” $\bar{x}_{cent}(p_i, T0_j, BPM_k)$, as a function of the “average beam radius” $\bar{r}_i(p_i, T0_j)$ and we plot the pairs of $\{\bar{x}_{cent}(p_i, T0_j, BPM_k), [\bar{r}_i(p_i, T0_j) - \bar{r}_0(p_0, T0_j)]\}$ as shown in Figure 4 below by black circles. The red rhombus shaped points in Figure 4 correspond to the value of the function which is used to fit of the experimental points. This function which fits the experimental points has been chosen to be a second order polynomial in $\delta\bar{r}$, $\bar{x}_{cent}(p_i, T0_j, BPM_k) = a\delta\bar{r} + b\delta\bar{r}^2$. The coefficient “a” of the first order term is proportional to the dispersion D_x .

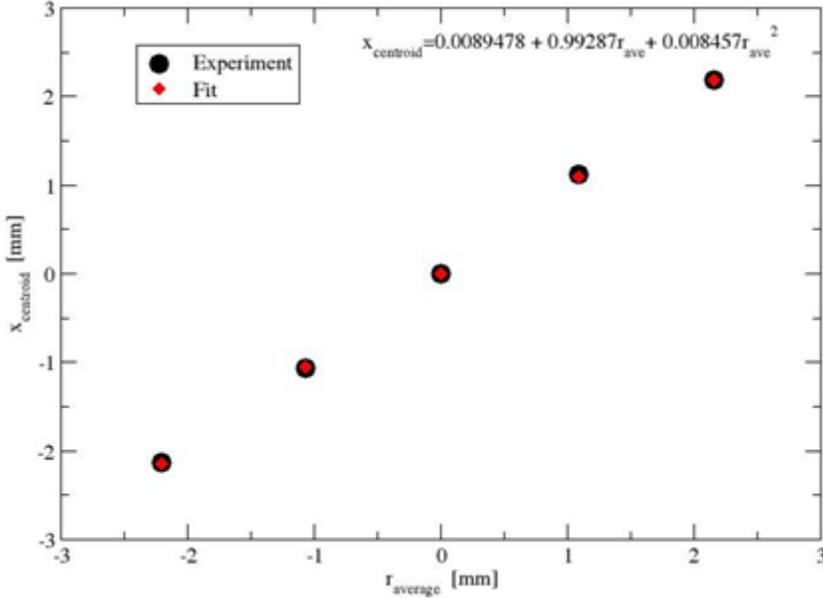


Figure 4. A plot of the experimentally measured position of the beam centroid x_{cent} as a function of the beam's average radius $\delta\bar{r} = \bar{r}_i(p_i, T0_j) - \bar{r}_0(p_0, T0_j)$ (black circles). The red circles is the fit to the experimental points using a second order polynomial in $\delta\bar{r}$, $\bar{x}_{\text{cent}}(p_i, T0_j, \text{BPM}_k) = a\delta\bar{r} + b\delta\bar{r}^2$. The coefficient of the first order term “a” is proportional to the dispersion D_x .

Since the average beam radius $\bar{r}_i(p_i, T0_j)$ is proportional to the beam momentum we can convert the 1st order coefficient “a” to the dispersion D_x by multiplying the coefficient by a factor 1.8 [mm/(dp/p)]. This factor appears on the third line of the data input file “dispersionFrom_AgsOrbitDisplay.in” discussed earlier, and can be changed before or after we collect the experimental data. This factor depends on the beam optics setup of the AGS, when the orbit data was taken. The calculated Horizontal and Vertical dispersions at the location of the BPM's for each $T0_j$ time are written on files under the names “Hor_Dispersion_xxxx” and “Ver_Dispersion_xxxx”. The “xxxx” symbols at the last part of the filenames correspond to the $T0_j$ times in msec. The calculated dispersions are also plotted by the use of the plotting program “gnuplot”. Figure 5 is an example of plotted dispersion at time $T0=572$ ms. The user may use the files “Hor_Dispersion_xxxx” and “Ver_Dispersion_xxxx” to plot again the horizontal and vertical dispersions with the plotting program of her choice. All files like “Hor_Dispersion_xxxx” generated by the calculations are written on the directory we executed the “AgsDispersion” computer code.

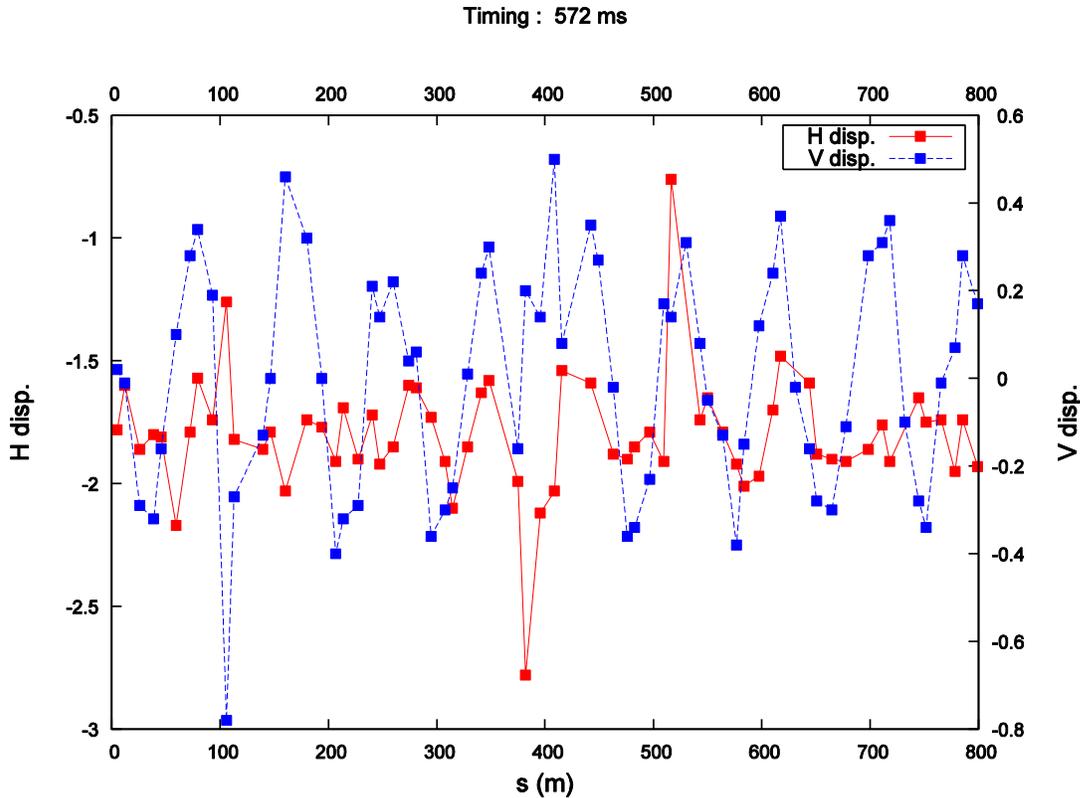


Figure 5. The Horizontal and vertical dispersions at the locations of the AGS BPM's as plotted with the plotting program gnuplot.

Reanalyzing the data

When reanalysis of the collected orbits is required, we only use the part of the code which calculates the dispersion from the measured orbits.

In this case we simply execute the command:

```
/operations/app_store/AgDispersionMeasure/dispersionCalculation/source/dispersionFrom_AgsOrbitDisplay
```

We should execute the command above at the same directory where the results from the first analysis of the data had been collected and analyzed for first time. The file “dispersionFrom_AgsOrbitDisplay.in” has been created in this directory after the data collection. This file contains the directory path of the filenames of the orbit files required for the reanalysis of the data. In case we want to reanalyze with a new factor which converts the average beam radius from mm to dp/p (see earlier section “*Running the code from a window*”) we change the value of this conversion factor in the third line of the file “dispersionFrom_AgsOrbitDisplay.in”.

Conclusions

An automatic computer code has been implemented to measure the dispersion function of the AGS at the location of the BPM's. The code has also been installed in the “StartUp” of the CAD control system in the form of an application under the name “AgDispersion”. The results of the measured dispersion

function can be used to test the online computer codes zgoubi and MADX which model the beam optics of the AGS. This automatic method of measuring the dispersion of the AGS not only reduces dramatically the time required to collect and analyze the experimental data to calculate the dispersion of the AGS, but also minimizes the possible human error that may occur in the process of the dispersion measurement.

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